Acoustics: Reflection Coefficients and Mapping

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Abstract

Every physical material has a specific acoustic reflection coefficient, which is what determines the strength of the reflection of a sound wave striking the material. This coefficient can be determined with a combination of simulation and experimental techniques. The experimental element of the project consists of transmitting and receiving sound waves in a certain environment. The environment is mapped and then a transmitter emits a sound wave of a specified frequency into the environment. The reflection of the sound wave off of boundaries such as the floor and the walls of the environment is then recorded with a receiver. This process creates an acoustic map of the environment. The data gathered from this experimental portion is then compared to a computer simulation of the experiment. The simulation, which is based on the Helmholtz wave equations, uses the boundary conditions of the environment and the parameters of the sound wave to produce a model of the wave reflections. If the appropriate reflection coefficient is used in the simulation it will produce results that match the experimental data. Thus, by matching the simulation to the data, the reflection coefficient of the materials of an environment can be determined.

I. Introduction

The motion of traveling waves is affected by the properties of the material through which it moves. When a wave moves from one medium to another, its motion changes in a predictable way[1]. Specifically, when encountering a boundary between two different mediums, the wave is both reflected off of the boundary and transmitted through the boundary, as illustrated in Figure 1. How much of the wave's energy is reflected and how much is transmitted depends on the properties of the materials through which the wave is traveling[1].

In the case of acoustic waves traveling through air, any object the wave comes into contact with acts as a boundary between two mediums. While part of the sound wave is reflected off the object, part of the energy of the wave is absorbed by the object as heat energy. The percentage of the wave's energy absorbed by the boundary is determined by the materials out of which the boundary is made[1]. Because a portion of the wave's energy is absorbed by the boundary, the initial wave that comes into contact with the boundary will be stronger than the wave that is reflected off the boundary. The ratio between the strength of the initial wave (the "incident wave") and the reflected wave is called the reflection coefficient[1]. Every material has a specific reflection coefficient and thus different materials have different abilities to either amplify or dampen sound[1].

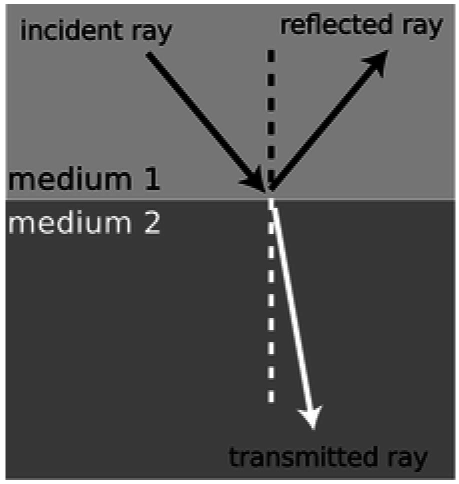

Figure 1. Model of transmitted and reflected rays from a single incoming ray.

This reflection coefficient is experimentally determined by measuring the strength of a known sound wave that has come into contact with a boundary. Our project utilizes two different techniques to experimentally determine the reflection coefficients of surfaces in a physical environment. The first technique was to directly measure the strength of a sound wave to determine the reflection coefficient and the second technique was to find the reflection coefficient by matching a simulation to recorded acoustic data. Once a reflection coefficient has been determined for all surfaces in an environment, it is possible to then create a model of the acoustics of a room, or select certain materials to shape the acoustics of a room as necessary.

II. Theoretical Background

The mathematical background of this project describes acoustic waves as sinusoidal plane waves. Although this not an entirely accurate description, the simplicity of working with plane waves make them an ideal model for our acoustic waves. The term "plane wave" refers to a wave of constant frequency that has wave fronts that are infinite parallel planes[2], as depicted in Figure 2. describe, in one equation, the magnitude, frequency, and direction of the wave.

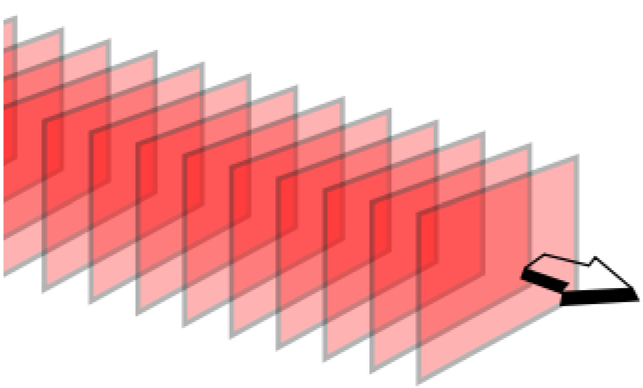

Figure 2. Depiction of a plane wave travelling in three dimensions.

The plane wave takes the form

$$u(\boldsymbol{r}, t) = M * e^{i(\boldsymbol{k} \cdot \boldsymbol{r})},$$

where M is the magnitude of the wave, $\boldsymbol{r}$ is the position vector $(x\ y\ z)$ and $\boldsymbol{k} = (k_x + k_y + k_z)$ is the wave number of the wave, and describes the direction of the wave's propagation[2].

Using this wave equation, we can determine how the wave will interact with a specific boundary, such as a wall or floor. The boundary is described using the equation for a plane,

$$Ax + By + Cz = D,$$

which holds for every point (x, y, z) in the specified plane[2].

III. Experimental Setup

To perform the experimental portion of the project, both a transmitting device and recording device were necessary. To transmit a sound wave, we used a cell phone which could emit sound waves, either constant or pulsed, at a chosen frequency. The recording device we used was a small microphone placed at the back of a wooden horn, as pictured in Figure 3. The microphone was connected to a computer which used audio software to record the incoming sound waves. The wooden horn was secured onto a tripod about three feet off the floor, and could be moved around a room relatively easily, a fact which we utilized in multiple aspects of our experiments.

Figure 3. The wooden horn, with attached microphone that served as a receiver for the sound waves.

The project consisted of two separate experiments conducted in different rooms on the American University campus. The first experiment took place in a theater room in the Katzen Arts Center.

The second experiment took place in Dr. Michael Robinson's office in the Mathematics Department located in Gray Hall. In order to accurately analyze the recorded sound waves, we had to record spatial measurements of both locations. These measurements are recorded in Figure 4.

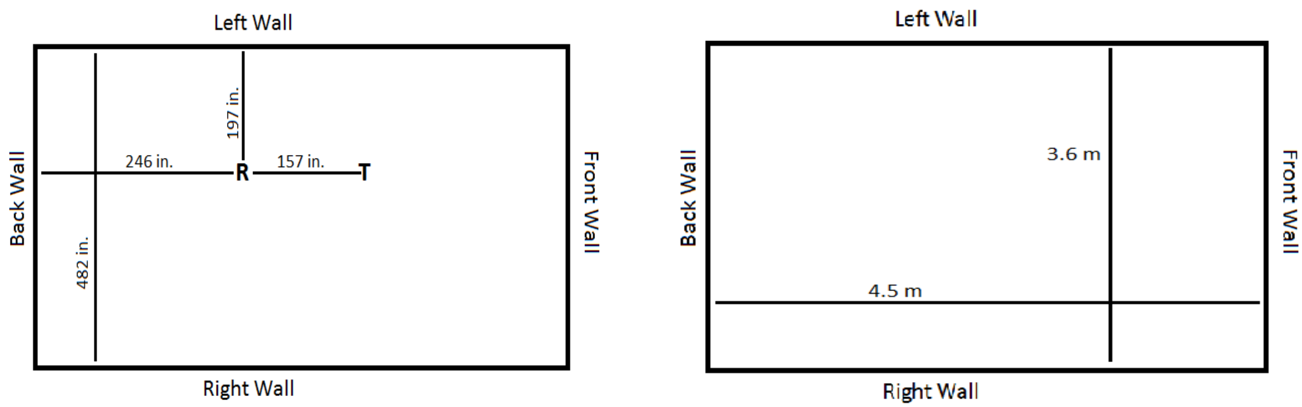

Figure 4. Schematics of theater room in Katzen (left) and Dr. Robinson's office (right).

Finally, before we performed our experimental tests, we had to record data about the antenna properties of the receiver. Since our receiver is not spherically symmetric, the level of signal it receives from the transmitter will change depending on which way the horn opening is facing. When it is facing directly towards the transmitter, for example, the receiver will record a higher signal level than if the side of the horn is facing the receiver if all other factors are held constant. This is the intended function of the horn: to direct sound waves incoming from the front of the horn and block waves coming in from the sides of the horn in order to reduce noise and error due to unwanted reflections of objects in the environment. To record this data, we set the transmitter to emit a constant sound wave and then rotated the receiver around 360°, recording the signal level for the continuous spectrum of degrees. The antenna pattern we recorded is shown in Figure 5. We used this data to determine how much of a decrease in signal level was due to the orientation of the transmitter/receiver setup.

IV. Simulation

Before starting any experimental aspects of the project, I first developed a simulation to model plane waves reflecting off boundaries in an environment. The program inputs are a single equation of a plane wave and a set of equations describing physical boundaries, as explained in the theoretical section above. In addition to the spatial description of the boundaries, however, the program further requires an input describing the reflection coefficient of the boundary. The program then sequentially finds the reflection of the plane wave over a single boundary and uses the inputted reflection coefficient to determine the strength of the reflection wave. It then uses this new, reflected wave as the incident wave on the second boundary, and continues this process for each inputted boundary equation. Once the wave has been reflected over each boundary, the

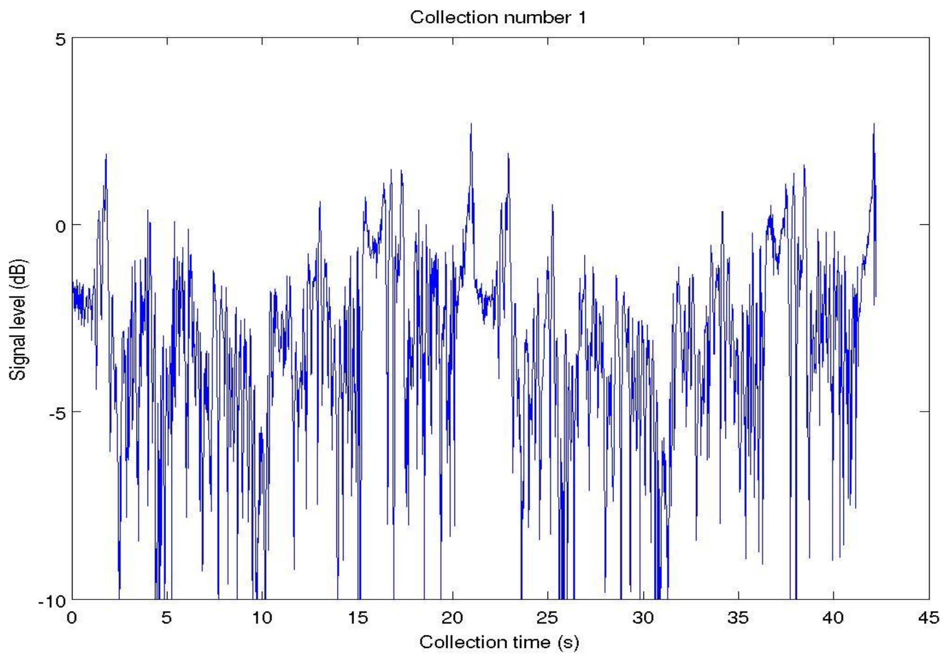

Figure 5. Antenna data shows the range in signal strength due to relative positioning of the receiver and transmitter.simulation returns the final reflected wave.

The program then creates a 2-D plot of the signal strength of the wave by location in the environment, as shown in Figure 6.

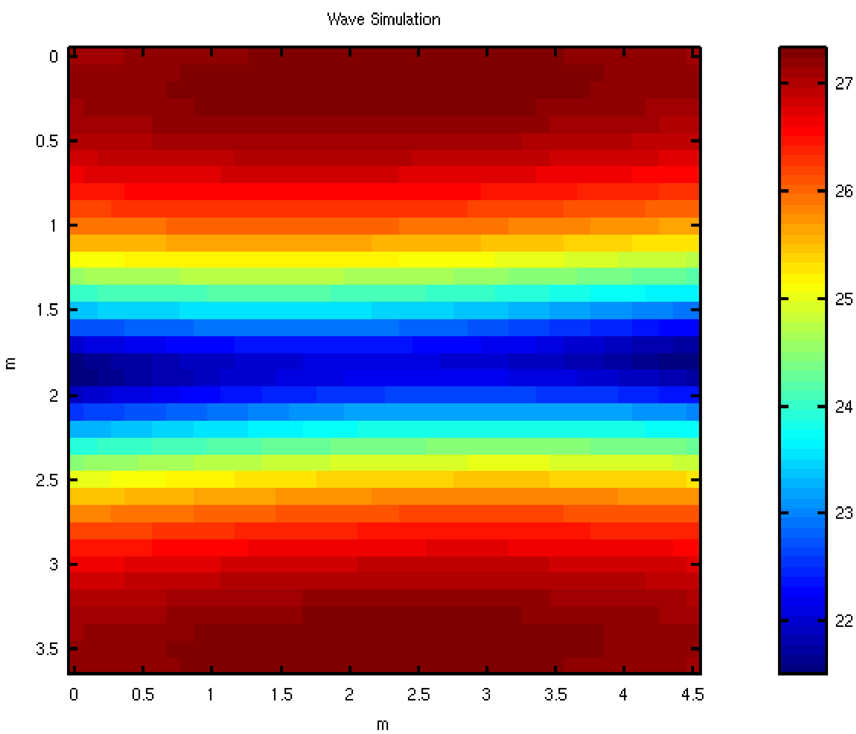

Figure 6.
Simulation of signal strength by poisiton in room.

V. Experimental Methods

Experiment 1

As mentioned earlier, the first experiment took place in a theater room in Katzen Arts Center. In this experiment both our receiver and transmitter remained stationary while the transmitter emitted a pulsed sound wave. The time between pulses (or period) of the sound wave was calculated to match the amount of time it would take the wave to travel from the transmitter to the receiver. This would help to ensure that we were only recording data from one wave pulse at a time, excepting reflections off other surfaces and miscellaneous objects in the room, which was generally unavoidable. The transmitter was positioned approximately 160 inches from the receiver, which was facing a single wall of the room. This wall was the focus of our data collection, and both the receiver and transmitter were positioned about 200 inches from the wall.

Experiment 2

Once again, this experiment was conducted in an office in Gray Hall. This experiment required the transmitter to emit a continuous sound wave while the transmitter remained in a stationary position. The position of the receiver, however, moved throughout the experiment. In the experiment, we performed two separate tests. In the first the receiver was oriented squarely facing a single wall of the room. The tests started with the receiver approximately 3.33 feet away from this wall. We then recorded the signal of the received wave at this position and then moved the receiver back several inches and recorded the signal strength again. This process was repeated 13 times, each time moving the receiver several inches farther away from the wall. Then, we shifted the receiver approximately a foot to the right, away from the transmitter and repeated the process, recording the signal strengths at the same distances from the wall but at this new distance from the transmitter. For clarification, the test locations are shown in Figure 7.

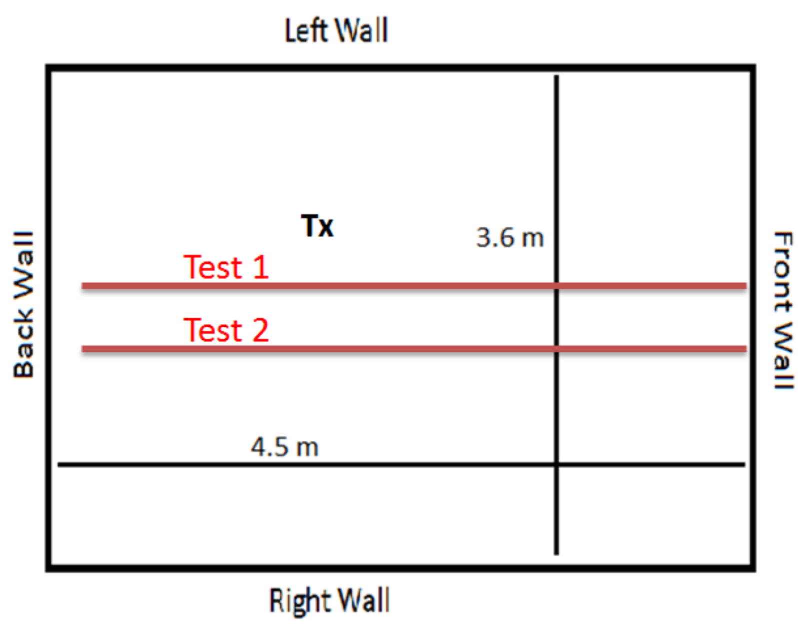

Figure 7. Map of test paths for Experiment 2. "Tx" indicates the location of the transmitter.

VI. Results and Analysis

Experiment 1

In experiment 1, we continuously recorded data while the sound pulses were being emitted. Figure 8 is a plot of signal strength over for each pulse period by distance from the receiver. Each vertical column of pixels represents a single wave pulse, and signal strength is indicated by brightness, with higher signal strengths brighter and lower signal strengths darker. Having multiple of these wave pulses lined up shows how the signal strength changes over time.

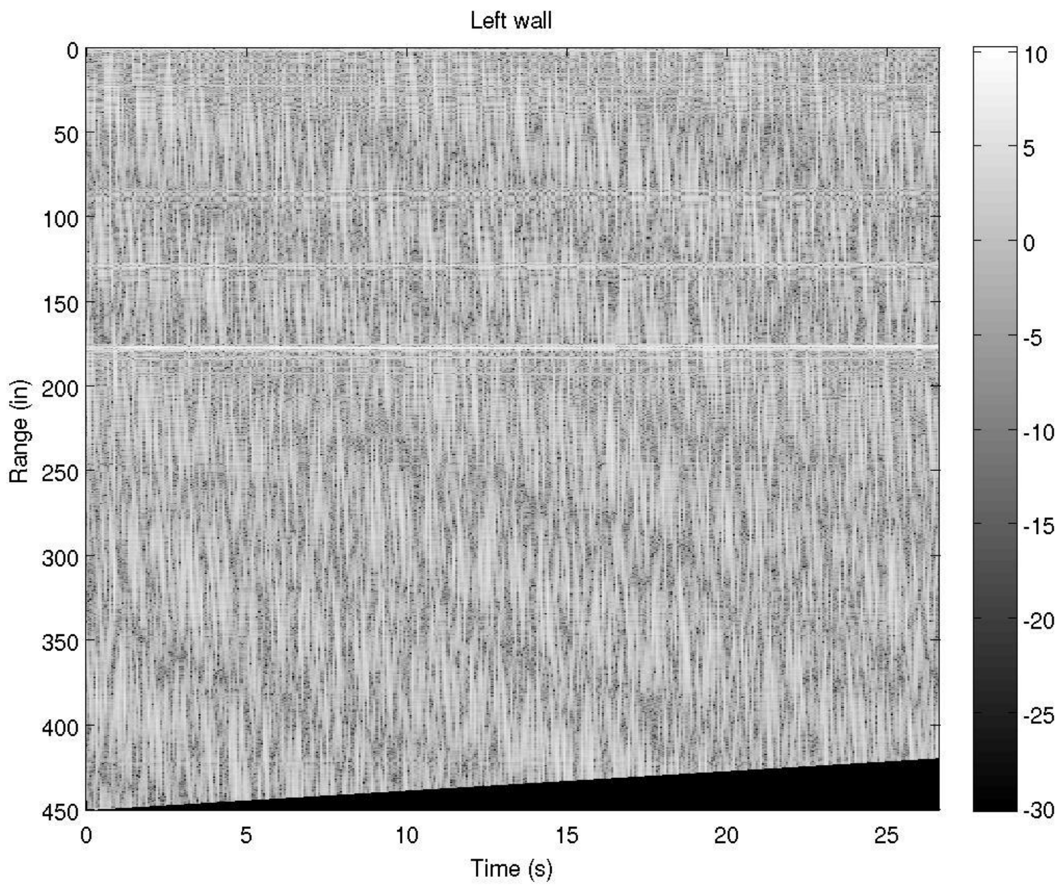

Figure 8. Plot of wave pulse signal strength over time.

Most of the plot is in the middle range of signal strengths, with a slight variation over the plot. However, there are three prominent, bright horizontal lines present. These horizontal lines mean that the receiver recorded a high signal strength from every wave pulse at close to the same distance from the wall. Since the signal is constant over time, this indicates that the sound wave was reflecting off a stationary object. When comparing the locations of these lines to objects in the room, we found that they correspond to the location of the wall the receiver is facing and two rows of chairs in front of the wall. In this way, the sound wave can be used to create a rough map of the environment in which it is travelling. By averaging the data across all wave pulses, we can compare the signal strength of the wave at each distance from the receiver. This is shown in Figure 9.

This figure is a plot of average signal strength by position from the receiver over the entire duration of the experiment. The three peaks shown correspond to the three horizontal lines from Figure 8, and therefore to the location of the wall and rows of chairs. By comparing the signal strength of these lines with the signal strength of the original wave (accounting for the antenna effect as mentioned earlier) we can determine the ratio of the signal strength of the incident wave to the reflected wave, which is the reflection coefficient. By comparing the signal strength of the peak corresponding to the wall, we find that the reflection coefficient of the wall is approximately 4 decibels (dB).

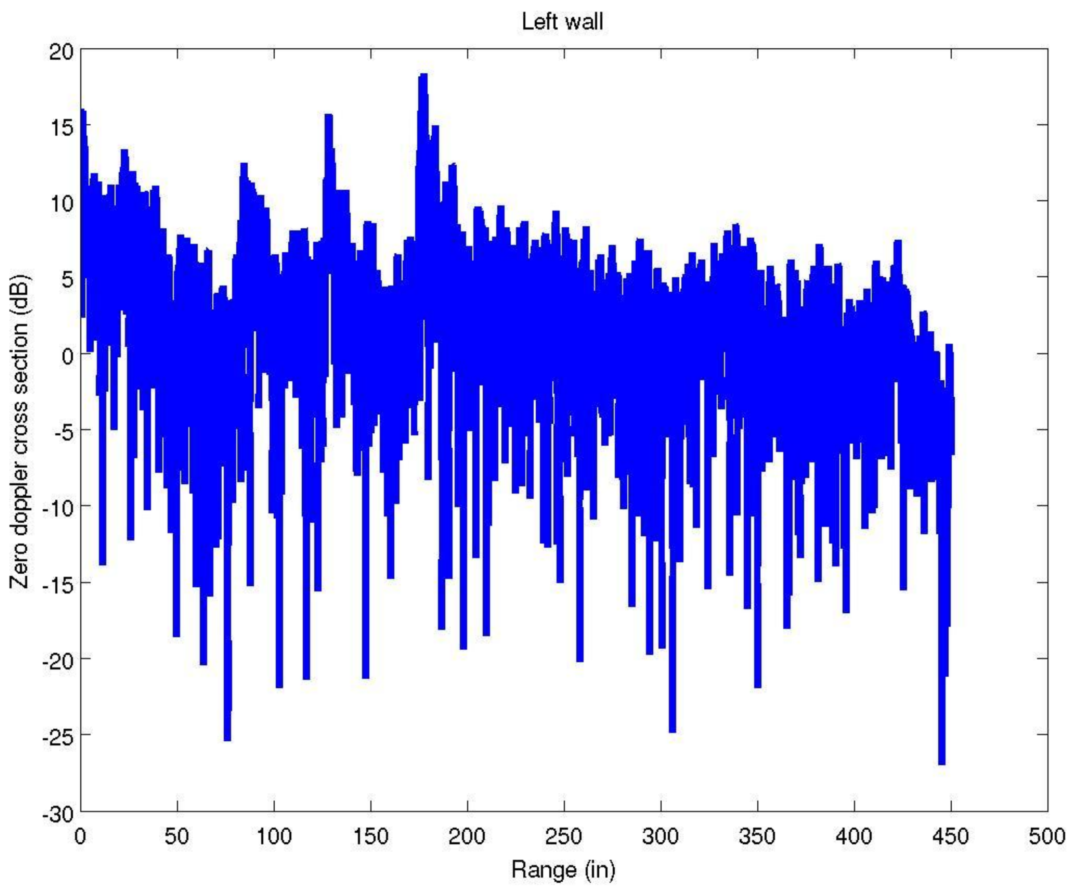

Figure 9. Plot showing signal strength of the sound wave (in decibels) versus the range in inches from the receiver.

This measurement of 4 dB is consistent with the reflection coefficient for a typical building material, confirming that our measurement and analysis techniques were successful and accurate.

Experiment 2

In our second experiment, we recorded signal strength in discrete intervals and thus made a plot of signal strength by distance from the wall for both tests performed, as shown in Figure 10.

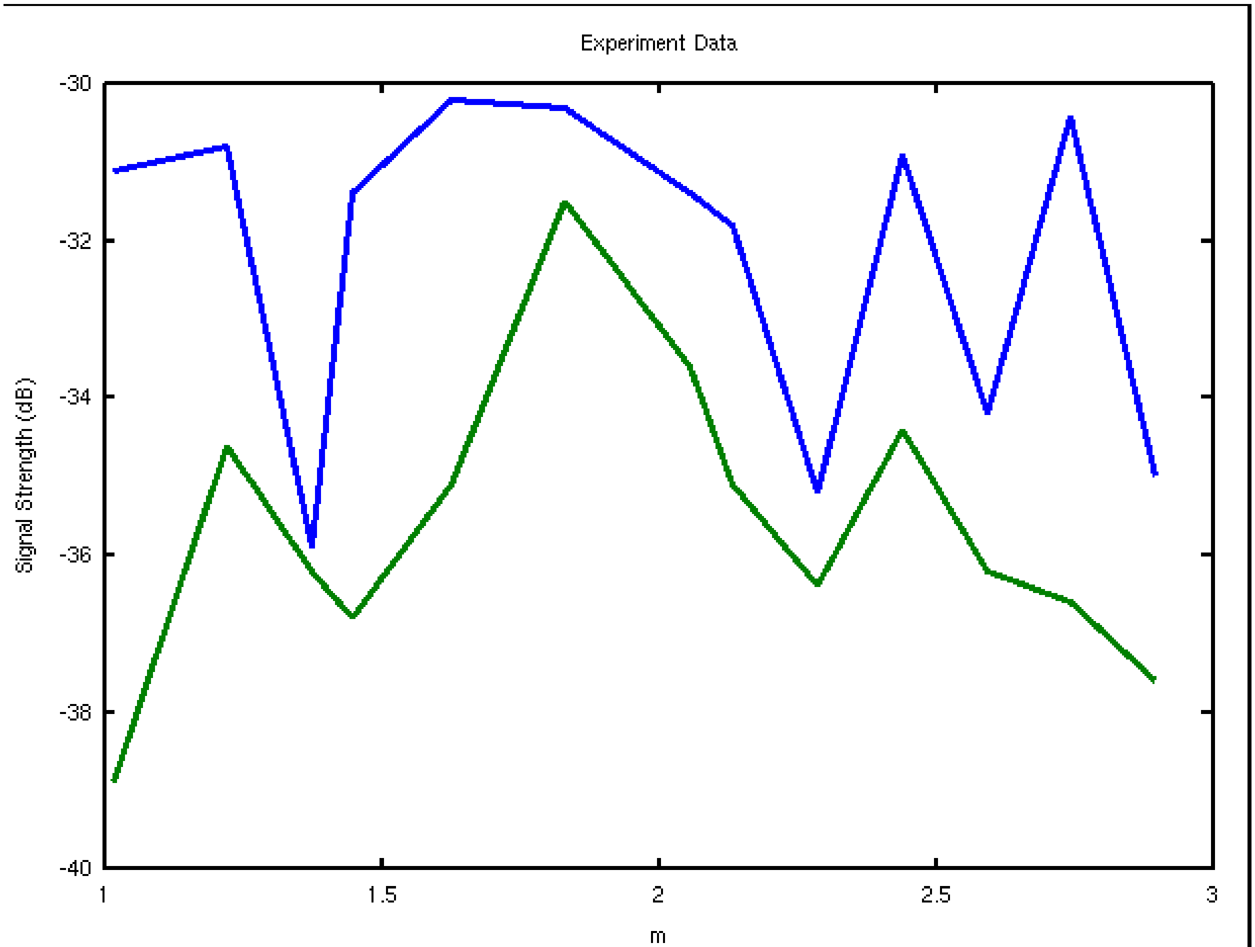

Figure 10. Plot of signal trength (dB) by distance from wall (m) for experiment two. The first test is represented by the blue line and the second test (with the receiver shifted a foot from the transmitter) is represented by the green line.

In this experiment, the simulation described earlier is used. Although the two tests (at different distances from the transmitter) have different signal levels, they follow a similar pattern in signal level variance and have a similar ratio between the highest and lowest signal strengths of about 6 dB. In order to find the reflection coefficient for the wall in this experiment, we utilized the Standing Wave Ratio (SWR), which is the difference between minimum and maximum signal strengths. By changing the inputted reflection coefficient for each boundary condition in the simulation, we attempted to match SWR of the signal strength in the data to the SWR of the signal strength in the plot created by the simulation. Using the data from this experiment, we found the reflection coefficient to be about 0.3, which once again is typical for usual building materials and thus again confirms our experimental methods.

VII. Conclusion

In conclusion, we found in this experiment that there are various ways in which to measure the reflection coefficients of materials in an existing environment, both purely experimentally and in conjunction with simulation techniques. Different methods may be suited for different environments or situations to overcome various sources of noise or error. Finding reflection coefficients have multiple application and while the techniques may be seem relatively simple, the theoretical background ensures that they are accurate and beneficial.

References

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2. Haberman, Richard. *Applied Partial Differential Equations: With Fourier Series and Boundary Value Problems*. 3rd ed. Upper Saddle River, NJ: Pearson Prentice Hall, 2004. Print.

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