The Effects of Regional Jets on Airline Scheduling Decisions

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Abstract. Flight delays impact both airlines and their passengers, resulting in wasted resources on the part of the firms and wasted time on the part of the customers. One of the potential causes of airport congestion is an increase in daily flights scheduled by airlines. This paper investigates the link between the recent popularity of smaller "regional jets" and airlines' flight frequency and aircraft size decisions. An agent-based simulation (ABS) is used to show that when regional jets are available in an airline's fleet, the airline will schedule more daily flights using smaller aircraft than if regional jets were not available. This may increase the surplus of customers who appreciate additional flight times, but could lead to additional delays. Two policy experiments intended to reduce congestion (departure taxes and slot controls) are found to reduce the number of daily flights scheduled in the simulation while also reducing producer surplus.

1 Introduction

Congestion in the national air traffic system imposes an enormous cost on businesses and individuals. A 2010 report by the National Center for Excellence for Airlines Operations Research (NEXTOR) reported that in 2007, congestion caused $32.9 billion in losses to the U.S. economy from delays and wasted time (Ball et al. 2010). The federal government has imposed several recent policies in an attempt to limit congestion, including a new fine on aircraft that sit on the tarmac for more than three hours before departing. The development of NEXTGEN, an updated air traffic control system that would also lower delays, has been plagued by cost-cutting, budget cuts, and bureaucratic red tape.

Meanwhile, the airline industry has been changing the configurations of its aircraft fleets to respond to rising and unpredictable fuel costs. The emergence of regional jets—small jet-powered aircraft that seat between 35 and 100 passengers—has revolutionized the ways in which airlines plan their routes. While regional jets are more expensive than other types of aircraft on a passenger-mile basis, their smaller size allows airlines to be more flexible in their scheduling and minimize wasted seats by responding more accurately to passenger demand.

However, the switch to regional jets on short-haul and medium-haul routes has exacerbated congestion problems in the nation's airports, since airlines using smaller aircraft must schedule more flights to accommodate demand. In response, policy-makers have implemented several congestion-reducing measures, including departure taxes and slot controls, in an attempt to limit the number of flights that depart per day from busy airports. The efficacy of these policies remains unclear.

This project explores the effects that regional jets have had on airlines flight frequency and aircraft size decisions. Using an agent-based simulation (ABS) model and operating-cost data from U.S. airlines, I show that the availability of regional jets causes airlines to reduce the aircraft size of a typical flight while increasing daily flight frequency in a market. In other words, airlines tend to use smaller regional jets to operate more daily flights between busy airports. This behavior can explain why these airports have seen rising congestion problems over the last decade.

I also use my simulation model to predict outcomes of several policy experiments. I find that removing regional jets from airline fleets, instituting a departure tax, and implementing a slot control on takeoffs and landings all succeed in reducing the number of daily flights operated in a given market. However, these policies also reduce total airline profits and may reduce consumer surplus as well, since consumers benefit from more frequent departure times. Future research should examine the cost-benefit analysis of these types of policies to see if the decrease in congestion warrants the resulting losses of surplus.

This paper begins with a review of previous studies in yield management and the limited literature surrounding regional jets. I then describe the development of my game-theoretic agent-based simulation model and explain why this approach is an appropriate one for modeling air travel markets. The model is then used to simulate Nash equilibrium outcomes in three hypothetical markets that are meant to replicate small, medium, and long-haul routes in the United States. Policy experiments are implemented in each of these markets, and the results are compared to the base case. I also use the model to attempt to replicate real-world outcomes in several actual U.S. markets. The successes and limitations of the model in accomplishing these tasks are discussed. A short discussion of possible future research concludes the paper.

2 Literature Review

The study of airline pricing, fleet size, and market entry decisions is often referred to as *yield management* or *revenue management*. Yield management focuses on how airlines can allocate their available resources (aircraft, personnel, and maintenance facilities) to maximize profits. Research in airline yield management can be grouped into three major categories: market entry decisions, pricing theory, and game-theoretic competitive analysis. This project relies on contributions in all three of these bodies of research, as well as cutting-edge advancements in agent-based simulations and computational economics, to investigate how the availability of regional jets changes airline decision making.

Market entry, or an airlines decision to start operating commercial flights between two cities (a *city-pair*), is one of the most heavily studied processes in airline yield management. The formal investigation of market entry can be traced to Berry (1992), whose seminal paper in *Econometrica* developed one of the first rigorous theoretical models of airline entry into a market. The Berry model finds a good deal of success in accurately estimating the number of entrants in various city-pair markets. However, the model only identifies the number of airlines that should enter a market, and not precisely which airlines will enter.

More recently, Boguslaski et al. (2004) and Ciliberto and Tamer (2009) have expanded upon the Berry model to allow for heterogeneity in airline characteristics when evaluating market entry decisions. Specifically, the profit functions and competitive advantages of large, "legacy carriers" like Delta Air Lines and United Airlines differ from those of smaller, "low-cost carriers" like Southwest Airlines and JetBlue Airways (Ciliberto and

Tamer 2009). This can be seen in the structure of these airlines' route networks—while legacy carriers usually operate using a traditional "hub-and-spoke" model, with passengers often required to make a connection through a central hub airport, low-cost carriers have generally developed their networks around a "point-to-point" model, in which a larger number of smaller "focus cities" serve as defacto hubs. Southwest Airlines is perhaps the best known example of an airline using such a point-to-point network (Yan et al. 2008).

The structure of airline networks can create various externalities for passengers and competing airlines. Hendricks et al. (1997) investigate these externalities using a game-theoretic framework. They find that regional airlines without a significant cost advantage in a spoke market will be deterred from entering, while larger airlines, even if they do not have a cost advantage, will stay in the market in order to keep some degree of market power. This may explain why large domestic carriers continue their hub-and-spoke networks even while Southwest Airlines, which operates on a point-to-point network, has been so successful. These types of differences in network structures, operating costs, and profit functions between various airlines can lead to more nuanced market entry decisions than the Berry (1992) model is able to predict.

Market entry analyses often go hand-in-hand with air travel pricing models. After all, accurate estimations of demand and ticket prices are critical in making successful market entry decisions. Airline tickets are priced in a highly complex and obfuscated manner. Airlines typically have a dozen or more "fare classes" that determine ticket prices. A customer purchasing a ticket is allocated a fare class based on her desired seating class (first-class, business, or economy), the length of her stay, whether she is buying a one-way or round-trip ticket, and the number of days between her purchase and the date of the flight, among many other factors. As yield management scholars Theodore Botimer and Peter Paul Belobaba (1999) note, these "differentiated fare products offered by airlines are targeted to distinct segments of the total demand for air travel in a market." That is, airlines create many classes of service in an attempt to capture the most amount of surplus from the air travel market—a clear example of price discrimination.

However, as Botimer and Belobaba make clear, this model of price discrimination assumes that airlines have full information about the characteristics of their passengers, and that a passenger takes no offense to being charged a different price than her seatmate on the same flight. In reality, air travel customers are highly price-sensitive and often have a remarkably clear picture of the fares available in a given city-pair market. The advent of internet-based self service travel services, as opposed to the traditional model of booking through travel agencies, has given customers more information about fares and flights. In response, airlines have started to make their pricing decisions by accurately predicting and matching their competitors decisions in each market (Vinod et al. 2009).

The reliance of airlines on each others decisions when setting fares creates a natural stage for game-theoretic analysis. Indeed, in the past five years, many papers have created a link between airline markets and the game-theoretic models of industrial organization. Much of this past research has focused on the relationships of network structure to airline entry decisions. For instance, Aguirregabiria and Ho (2009) use game theory to explore whether existing hub-and-spoke networks act as a deterrent to new firms entering a given market. The authors also estimate the start-up costs for an airline to enter a new market. They find some evidence that hub-and-spoke networks create entry deterrence, but conclude that “the net welfare effect of this type of entry deterrence behavior is ambiguous.” Other papers (Isler and Imhof 2008, Netessine and Shumsky 2005) have used game theory to explore airlines pricing decisions once market entry has already occurred.

However, the existing literature provides little insight into the ways in which airlines make decisions about flight frequency in a given city-pair market. Much of the previous market entry research has only focused on whether an airline should enter a market or not—no attention is paid to the how many daily flights should be offered once entry occurs. Another critical airline decision that has been underexplored in the literature is the choice of aircraft size. The past decade has seen a rise in smaller jet aircraft that typically seat 50-100 passengers in many routes that were traditionally operated using either larger jet aircraft or smaller turboprop planes. The mainstream availability of these smaller “regional jets” have drastically changed the ways that airlines make decisions, yet the impacts of regional jets on producer and consumer have only been studied in a handful of papers.

Most of the limited research on regional jets has an econometric flavor. As regional jets started to gain popularity in the early 2000s, twin studies by Babikian et al. (2002) and Dresner et al. (2002) examined the cost structures of these smaller aircraft and how legacy carriers were using regional jets in their existing networks. Perhaps surprisingly, these studies found that regional jets are actually more expensive to operate on a seat-mile basis than larger narrowbody aircraft (Babikian et al. 2002). This means that regional jets must offer an advantage to airlines besides cost. Indeed, in a study of Continental Airlines, Dresner et al. (2002) find that Continental uses regional jets not to replace service that is already operated by turboprop aircraft, but instead to add new routes to farther-flung destinations or increase flight frequency in dense markets. The authors predict that the use of regional jets will result in congestion in hub markets—a theme that will be further explored in this paper.

The links between aircraft size and flight frequency decisions have been only recently explored. Brueckner and Pai (2009), following an earlier paper by Brueckner (2004) on airline scheduling and frequency decisions, investigate the impacts of the introduction of regional jets into a hub-and-spoke airline network. Particularly, the authors show that introducing smaller capacity jets into the market allows the airline to offer, in theory, more frequent flights. This increases consumer welfare, since flight frequency is one of the factors of consumer utility for air travel. In agreement with Dresner et al. (2002), Brueckner and Pai predict that smaller jets will result in new point-to-point routes that were not economically feasible to operate with larger aircraft. However, they find little empirical evidence to back up this “new routes hypothesis.”

Following from the theoretical analysis of Brueckner and Pai (2009), Pai (2010) examines some empirical factors that lead airlines to increase flight frequency or use smaller regional jets. Specifically, Pai finds that an increase in managerial workers or persons under 25 years of age results in an increase of both flight frequency and the use of smaller aircraft. Increases in wealth and in population have similar effects. Pai argues that capacity should not be constrained to reduce delays, but rather a system of congestion pricing should be implemented that allows time-sensitive passenger to pay for the privilege of departing when they choose.

The rising popularity of regional jets has accompanied a fundamental shift in the structure of the air travel market. While legacy carriers like American Airlines, Delta Air Lines, and United Airlines are scheduling regional jet flights at an unprecedented rate, these legacy carriers are not operating the flights themselves. Instead, smaller “regional airlines” have been formed to operate flights using regional jets. These airlines, which include such unfamiliar names as Comair, Chautauqua Airlines, SkyWest Airlines, ExpressJet Airlines, and Republic Airlines, operate flights on behalf of the legacy carriers. The regional airlines aircraft are painted with the legacy carriers logos and liveries, and customers are often unaware that they are flying on a regional airline that is often financially separate from the legacy carrier.

Some of these regional airlines are owned by the legacy carriers for whom they operate. For instance, Comair is owned by Delta Air Lines and American Eagle Airlines is owned by American Airlines. Other regional airlines are owned by publicly-traded holding companies that are financially separate from the legacy carriers. These airlines form contracts with legacy carriers to operate flights on the mainline airlines behalf. These diverse relationships between legacy carriers and their regional affiliates can result in different market outcomes. Forbes and Lederman (2009) investigate some of these links between ownership of regional airlines and the usage patterns of those airlines. They find that there is a strong link between the usage of owned regional airlines and markets in which there are lengthy delays or frequent snowfall, or between hub or focus cities. The authors postulate that by vertically integrating regional carriers, major airlines can streamline their operations and minimize “suboptimal” delay management decisions.

With the recent rise of regional airlines, the modern air travel market has become more nuanced and complex than the days of Berry (1992). The number of players in the market and their frequent interactions seem to beg for a computational approach. Agent-based simulation (ABS) is a computational technique that allows social scientists to model the relationships and strategic interactions of different players (“agents”). ABS techniques allow for elegant examinations of problems that would be too difficult to solve discretely, and allow for easy implementation of experiments that would complicate traditional mathematical analysis (Isaac 2009).

Very few previous papers have used a simulation to model the air travel market. Wei and Hansen (2007) provide one of the first examples of marrying game-theoretic analysis to a computational simulation to predict market results. Wei and Hansen (WH) develop three game-theoretic models of aircraft choice. The authors first derive cost functions for a variety of different airlines. They then use their game-theoretic models to simulate the aircraft choice and flight frequency decision for two identical airlines entering the Albuquerque, NM Phoenix, AZ (ABQ-PHX) market. The results from all three models show that firms choose the smallest possible aircraft and vary the flight frequency based on changes in demand.

Vaze and Barnhart (2011) also use an agent-based simulation to predict airline decision-making, with a particular focus on slot-controlled airports. By solving a constrained optimization equation for each airline (each agent), the authors are able to predict the flight frequency decisions at airports in which the number of takeoffs and landings (“slots”) is limited for each airline. Since airlines decisions about flight frequency are in part based on their competitors’ decisions, Vaze and Barnhart’s analysis is a game-theoretic one. In a very recent paper, Evans and Schäfer (2011) also use game-theoretic analysis to simulate an airline decision-making game. The Evans-Schäfer (ES) model differs from the Vaze-Barnhart (VB) model in methodology, but comes to a similar conclusion to both VB and WH in that capacity-controlled airports like Chicago’s O’Hare International Airport will become increasingly congested and crowded unless capacity expansion occurs in the near future.

My paper follows in the spirit of the Wei-Hansen, Vaze-Barnhart, and Evans-Schäfer models to explore a question left unstated by all three: do regional jets themselves cause airlines to increase flight frequency and reduce aircraft size? My model is perhaps most similar to the Wei-Hansen analysis, as I examine Nash equilibria flight frequency, aircraft size, and profit outcomes in several theoretical markets. However, unlike

Wei and Hansen, I use empirical cost data from U.S. airlines to attempt to predict actual market outcomes with specific aircraft. Additionally, I conduct policy experiments to remove regional jets, impose a departure tax, and institute a slot control to examine whether regional jets or capacity controls impact flight frequency decisions and congestion. I also explore the impacts of these congestion-reduction policies on producer surplus. These policy experiments are distinct from the three previous simulation models, which focused on airports in which controls were already in place and did not explicitly explore the impact of regional jets on the decision-making process.

3 Model Development

Let A be a set of airlines in city-pair market m. For each airline $a \in A$, we want to find:

$$\underset{K_a, F_a}{\text{argmax}}\ \pi_a = (p_m * (K_a * LF) * F_a) - (\alpha_{a,K_a} * \phi_m * K_a * F_a) \tag{1}$$

subject to the following constraints:

$$\sum_a K_a * F_a * LF \leq D_m \tag{2}$$

$$K_a * F_a * LF \leq \frac{F_a}{\sum_{-a \in A} F_{-a}} * D_m \tag{3}$$

Equation (1) describes a standard profit maximization problem. Constraint (2) ensures that the sum of all filled seats is less than the total demand in the market. Constraint (3) ensures that each airline's filled seats ($K_a F_a$*LF) do not exceed its market share, defined here as the ratio of airline a's flight frequency to the total flight frequency of all other airlines. Constraint (3) follows from Vaze and Barnhart (2011).

Variable	Description
D_m	Daily market demand (# of passengers) in market m
p_m	Average fare in market m
LF	Load factor
K_a	Aircraft size for airline a
F_a	Flight frequency for airline a
α_{a,K_a}	Cost per available seat mile (CASM) for airline a and aircraft K_a
ϕ_m	Distance between cities in market m

Table 1: Description of variables in equations (1) - (3)

Note that each airline's profits in equation (1) depend not only on their aircraft size and flight frequency decisions, but also on the decisions of their competitors. This profit-maximizing decision is therefore game-theoretic in nature. We are looking for one or more Nash equilibria–steady states in which no airline in the market would choose to change their flight frequency or aircraft size decisions even if they have perfect information about their competitors' decisions. There may be zero, one, or multiple Nash equilibria in any given market.

While computing Nash equilibria manually can be difficult, an agent-based simulation (ABS) is an ideal tool for simulating possible outcomes. Agent-based modeling refers to the process of programming various actors, or "agents," each of whom are acting according a set of predefined preferences or rules. In this case, the agents in the model are the airlines, each of whom can select a flight frequency and aircraft size from their set of scheduling possibilities while maximizing their profit function. Agent-based simulations can easily allow for heterogeneous agents, a condition that is often difficult in theoretical analysis. The simulation used in this paper was programmed in Python 2.6.6.

Demand data and other parameters for the model were collected from 2010 data available from the Bureau of Transportation Statistics (BTS). The table below summarizes the data sources used in this project.

Table 2: Data sources for variables and parameters

Variable	Description	Proposed Source
D_m	Mkt. Demand	BTS Form 41 T-100 Domestic Segment (U.S. Carriers) dataset
p_m	Avg. Fare	BTS DB1B Coupon Database and FlightAware Airline Insight
LF	Load Factor	BTS Form 41 T-100 Domestic Segment (U.S. Carriers) dataset
K_a	Aircraft Size	Seating information (# of seats) from airline websites
F_a	Frequency	Integer values 0 through 30 inclusive
α_{a,K_a}	CASM	Calculated using ASM data from BTS Form 41 Schedule T2 and aircraft-specific operating cost data from BTS Form 41 Schedule P-5.2
ϕ_m	Distance	BTS Form 41 T-100 Domestic Segment (U.S. Carriers) dataset

Revenue Calculations

To calculate the revenue portion of the airlines' profits functions, I used market demand data from the 2010 Bureau of Transportation Statistics Form 41 T-100 Domestic Segment dataset. This dataset reports the number of scheduled and operated flights between each city-pair in the United States on a monthly basis, along with the number of passengers that flew that route in the given month. I summed the total number of passengers that flew each month in each market during 2010 and divided by 365 to calculate daily demand passenger demand in that market.

I consider airline choices of aircraft and flight frequency in three theoretical markets and three empirical markets. The choices of markets that I have made are arguably arbitrary. However, since the three empirical markets that I have selected are all duopolistic, they are well-suited for use in the simulation, as described below. The markets are listed in Table 3.

Table 3: Selected City-Pair Markets

Airport 1	Airport 2	Market Size	Operating Airlines (2011)
Portland, OR (PDX)	Seattle, WA (SEA)	453,994 pax	Horizon, SkyWest
New York, NY (LGA)	Washington, DC (DCA)	416,586 pax	Shuttle America, US Airways
Boston, MA (BOS)	Atlanta, GA (ATL)	686,493 pax	AirTran, Delta

The simulation produced the most tractable results in markets with only two airlines competing, so the three markets selected for the empirical portion of the study are all duopolistic. It appears that duopoly markets are more likely to occur between cities that are close together (this would be another interesting research project!), so smaller markets may be overrepresented in this study.

Cost Calculations: Cost per Available Seat-Mile (CASM)

To calculate the costs of the airlines' operations, I used an accepted industry standard: Cost Per Available Seat-Mile (CASM). This measure calculates the cost to fly one available seat one mile. If the available number of seats in an aircraft and the flight distance is known, CASM data can be used to calculate the operating cost for a flight.

However, despite its use in the industry, CASM data was not readily available. In order to estimate airline costs, I calculated the CASM using Available Seat-Mile (ASM) data from the BTS Form 41 Schedule T2 dataset and operating cost data from the BTS Form 41 Schedule P-5.2 dataset. Operating cost varies from airline to airline and from aircraft to aircraft, so CASM was calculated individually for each airline and aircraft in the study using 2010 data.

To complete the study, I calculated CASM data for 17 U.S. airlines. These airlines represent a mix of older "legacy carriers" (e.g. Delta and United), newer "low-cost" carriers (e.g. JetBlue and Southwest) and regional carriers (e.g. Mesaba and Shuttle America). The airlines included in the study, along with their two-letter IATA Airline Designation codes, are summarized in Table 4.

Table 4: Airlines included in Study (with IATA Airline Designators)

Air Wisconsin (ZW)	JetBlue Airways (B6)
AirTran Airways (FL)	Mesaba Airlines (XJ)
American Airlines (AA)	Republic Airlines (9E)
American Eagle Airlines (MV)	Shuttle America (S5)
Atlantic Southeast Airlines (EV)	SkyWest Airlines (OO)
Delta Air Lines (DA)	Southwest Airlines (WN)
ExpressJet Airlines (XJ)	United Airlines (UA)
Frontier Airlines (F9)	US Airways (US)
Horizon Air (QX)	

For each airline, I calculated CASM data for every aircraft in the airline's fleet. In all, I made 88 CASM calculations. Aircraft were divided into categories based on their relative size. Aircraft with 30-100 seats were classified as regional jets, aircraft with 100-160 seats were classified as small narrowbodies, and aircraft with 160-220 seats were classified as large narrowbodies. Aircraft with two or more aisles are commonly referred to as "widebodies" and were classified accordingly.

The following tables summarize the cost calculations described above. Table 5 describes the types and classifications of aircraft included in the study. Table 6 is a summary table of aircraft capacity by aircraft type for the 17 airlines included in the study. Finally, Table 7 provides descriptive statistics about CASM for each of the four categories of aircraft. These CASM data will later be used as parameters for the generic airlines in our hypothetical model. Figure 1 provides a graph of the data in Table 7.

Table 5: Aircraft by Aircraft Type

Regional Jets	Small Narrowbodies	Large Narrowbodies	Widebodies
Canadair CRJ-100	Boeing 717	Boeing 737-800	Boeing 747
Canadair CRJ-200	Boeing 737-300	Boeing 737-900	Boeing 767
Canadair CRJ-700	Boeing 737-400	Airbus A321	Boeing 777
Canadair CRJ-900	Boeing 737-700	Boeing 757-200	Airbus A330
Embraer ERJ-135	Airbus A319	Boeing 757-300	Airbus A340
Embraer ERJ-140	Airbus A320		Airbus A380
Embraer ERJ-145	McDonnell-Douglas MD-88		
Embraer E-175	McDonnell-Douglas MD-90		
Embraer E-190	McDonnell-Douglas DC-9		
Embraer EMB-120			

Table 6: Aircraft Capacity by Aircraft Type

Aircraft Type	Min	Max	Mean	St. Dev	N
Regional Jet	37	100	61.5	18.3	25
Small Narrowbody	117	162	135.0	13.0	33
Large Narrowbody	160	224	184.7	22.3	10
Widebody	168	393	266.0	59.8	20

Source: Airline websites

Table 7: Cost per Available Seat Mile (CASM, in Dollars) by Aircraft Type

Aircraft Type	Min	Max	Mean	St. Dev	N
Regional Jet	0.028	0.213	0.094	0.054	25
Small Narrowbody	0.053	0.119	0.077	0.017	33
Large Narrowbody	0.038	0.073	0.058	0.013	10
Widebody	0.041	0.119	0.073	0.022	20

Sources: BTS Form 41 Schedules T2 & P5.2 (2010). Own calculations.

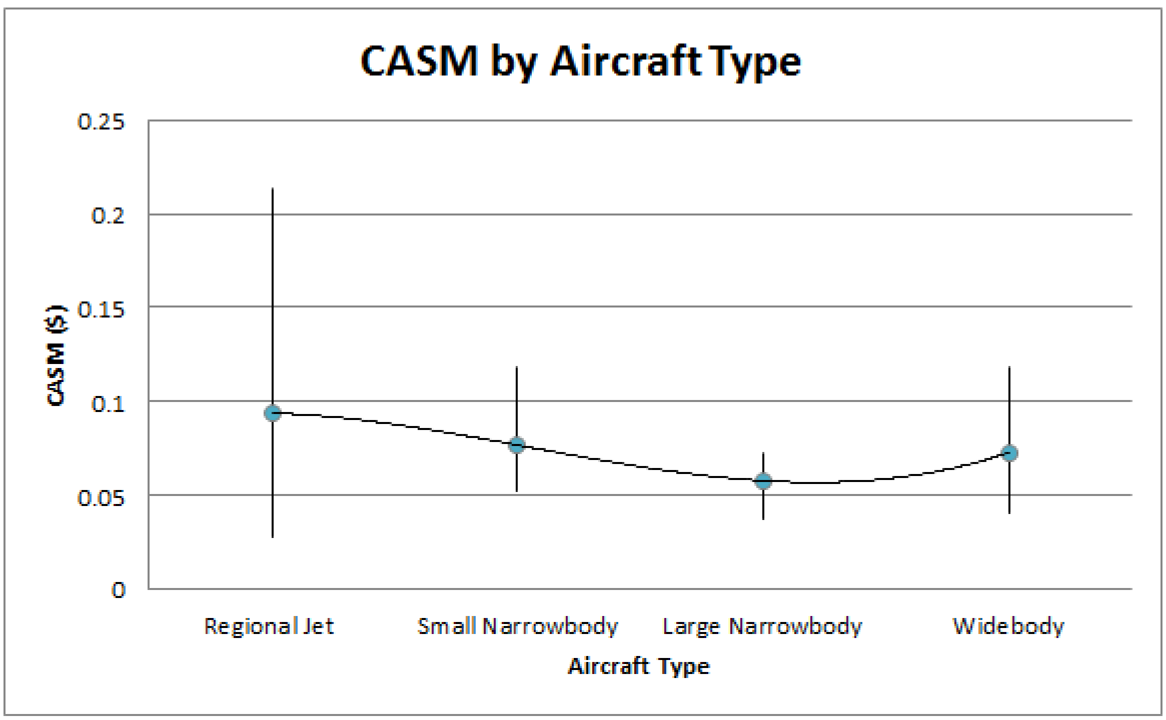

Figure 1: CASM on Aircraft Size

Testing the Hypothesis

I test my hypothesis using my agent-based simulation in two stages. First, I calculate Nash equilibria in three generic, theoretical markets in which two identical airlines compete. The characteristics of these identical airlines are derived from the mean calculations of cost per available seat mile (CASM) and aircraft size from the 17 airlines summarized in Table 4.

Once the Nash equilibria are calculated, I implement three policy experiments designed to ease congestion by reducing the number of flights in the market. The policy experiments include the removal of regional jets from the airline fleets, the institution of a departure tax, and the imposition of a slot control. I then compare the results of these experiments to the baseline equilibria to test my hypothesis that the availability of regional jets will result in airlines increasing flight frequency while reducing airline size.

Next, I test the model's real-world validity by simulating airlines' competitive actions in three city-pair markets. Due to limits in computational speed and ease of analysis, I have limited these markets to those in which only two airlines compete. I compare the simulated equilibria to the actual flight frequency and fleet configuration decisions that the airlines made in 2010.

4 Results: Theoretical Models

Market and Airline Characteristics

In the theoretical model, I simulate over three representative markets using two identical airlines. The characteristics of each of the three markets are summarized in the table below.

Table 8: Generic Market Characteristics

Market	Description	Market Demand	Load Factor	Avg. Fare	Distance (mi)
A	Small	300000	0.8	$100	250
B	Medium	600000	0.8	$175	1000
C	Large	1200000	0.8	$250	3000

The characteristics of these markets are based on observations of small, medium, and large-sized markets in the BTS datasets. Changing the parameter values will result in slightly different flight frequencies, but the aircraft size selection and the overall results of the analysis remain the same without loss of generality.

The theoretical model operates using two identical airlines. The airlines each have a choice of four generic aircraft types: Regional Jets, Small Narrowbodies, Large Narrowbodies, and Widebodies. The cost per available seat mile (CASM) and aircraft size for each aircraft type are same for each airline. The CASM and aircraft size parameters are equal to the means of the 2010 operating cost data summarized in Tables 6 and 7. The 17 airlines covered in this study represent a mix of regional carriers, legacy carriers, and low-cost carriers, so the mean data are representative of a typical airline in the US air travel market. The table below summarizes the CASM and aircraft size parameters for the generic airlines.

Table 9: Generic Airline Characteristics

Aircraft Type (K_i)	CASM	Size (# of Seats)
Regional Jet (RJ)	0.094	62
Small Narrowbody (SNB)	0.077	135
Large Narrowbody (LNB)	0.058	185
Widebody (WB)	0.073	266

Note that, as shown in Figure 1, the CASM data suggest a U-shaped relationship between aircraft size and number of seats. On a per seat-mile basis, the regional jets are the most expensive out of the four airline types, and the large narrowbody jets are the least expensive. However, the smaller size of regional jets offers a level of flexibility that allows airlines to capture the maximum amount of surplus, making regional jets the aircraft of choice for most routes in the simulation.

Theoretical Model Results

The table below presents the results of the simulation for the three generic markets. In the table, K_i represents the aircraft choice for airline i, F_i represents the daily flight frequency choice for airline i, and π_i represents the daily profit for airline i in dollars for the specific market. The table below, along with the other tables of results in the paper, presents the Nash equilibria generated by the simulation for each of the three generic markets.

Table 10: Theoretical Market Simulation Nash Equilibria (Base Case)

Market	K_1	F_1	π_1	K_2	F_2	π_2
A (Small)	LNB	4	\$48,470	RJ	4	\$13,826
	SNB	5	\$41,006	RJ	5	\$17,283
	RJ	8	\$27,652	RJ	8	\$27,652
	RJ	5	\$17,283	SNB	5	\$41,006
	RJ	4	\$13,826	LNB	4	\$48,470
B (Medium)	LNB	6	\$91,020	SNB	6	\$51,030
	SNB	6	\$51,030	LNB	6	\$91,020
C (Large)	LNB	11	\$52,910	LNB	11	\$52,910

K_i = Aircraft choice for airline i, F_i = Flight frequency for airline i, π_i = Profit for airline i

Remark. Let K_a and K_b be aircraft choices and F_a and F_b be flight frequency choices, and let Airlines 1 and 2 have identical costs. Then the action pair $[(K_a, F_a),\ (K_b, F_b)]$ is a Nash equilibrium in this model if and only if $[(K_b, F_b),\ (K_a, F_a)]$ is a Nash equilibrium.

It is also interesting to note that even though the regional jet option often does not provide the highest level of *individual* profits for the airlines, it is often listed as a best response, particularly in smaller markets. Indeed, in all five of the equilibria in the small market, at least one airline chooses to fly regional jets. This may suggest that the availability of regional jets creates a Prisoner's Dilemma situation, in which firms always have the incentive to choose regional jets and offer more flights than their competitors, even if this decision is less profitable *ex post*.

Since airlines are only allowed to choose one type of aircraft in this model, it is possible that regional jets are chosen more frequently because their smaller size allows airlines to minimize the number of wasted seats. For instance, if the daily demand in a given city-pair route was 1000 passengers, an airline would prefer to schedule 20 regional jet flights that seat 50 passengers each, resulting in 0 empty seats, instead of 8 small narrowbody flights that seat 135 passengers each, resulting in 80 empty seats. This reasoning could explain why regional jets are chosen so often in the simulation.

Experiment 1. Removal of Regional Jets

To test my hypothesis that the availability of regional jets causes airlines to choose more frequent flights and smaller aircraft sizes on average, my first policy experiment removes regional jets from the airlines' choice sets. Airlines can now only decide between small narrowbodies, large narrowbodies, and widebodies. The table below presents the equilibria that result from the simulation of Experiment 1.

Table 11: Theoretical Market Simulation Nash Equilibria (Experiment 1: No Regional Jets)

Market	K_1	F_1	π_1	K_2	F_2	π_2
A (Small)	LNB	3	\$36,352	SNB	3	\$24,603
	SNB	3	\$24,603	LNB	3	\$36,352
B (Medium)	LNB	6	\$91,020	SNB	6	\$51,030
	SNB	6	\$51,030	LNB	6	\$91,020
C (Large)	LNB	11	\$374,810	LNB	11	\$374,810

K_i = Aircraft choice for airline i
F_i = Flight frequency for airline i
π_i = Profit for airline i

Comparing these results to the equilibria in Table 10 seems to confirm my hypothesis. The small market is the most vivid proof: when regional jets were available, the equilibria resulted in each airline flying as many as eight regional jet flights per day. When regional jets are removed, there are now only two equilibria: each with one airline flying three large narrowbody flights and the other flying three small narrowbody flights. This reduces the total number of departing flights from as many as sixteen to six.

However, the removal of regional jets seems to lead to a more unequal distribution of profits between the two firms. Again, the small market provides the most vivid example. With regional jets, both firms made the same profit of $27,652 per flight, resulting in a total profit of $55,304. When regional jets are removed, one firm makes a profit of $36,352, while the other firms makes $24,603 per flight. The total profit is now $60,955. It is interesting to note that total profit actually increases in the small market when regional jets are removed. However, removing regional jets will likely decrease consumer surplus, since customers value the more frequent flight schedules that regional jets provide (Brueckner and Pai 2009).

Experiment 2. Departure Tax

Another method for reducing airport congestion is the institution of a departure (or landing) tax. This is a fee that airlines pay to the airport for each departure. By instituting this tax, airports hope to shift the airline supply curve up and to the left, resulting in fewer departures. However, setting the departure tax too high might lead to the airline deciding to withdraw from the market or relocate to a neighboring airport, robbing the original airport of any surplus.

To test whether departure taxes cause airlines to increase airline size and reduce flight frequency, per-flight departure fees were deducted from airline profits in the three generic markets. Regional jets were reintroduced into the airline fleets, so airlines could once again choose between regional jets, small narrowbodies, large narrowbodies, and widebodies when making their aircraft decisions.

Table 12 summarizes the results from Experiment 2. Instituting departure taxes often increased the number of equilibria in the market, so results are presented showing the number of equilibria, the largest number of daily total flights ($F_1 + F_2$) among all equilibria, and the largest total daily profit ($\pi_1 + \pi_2$) among all equilibria. Results are reported by market size and by departure fee per flight.

These results confirm the hypothesis that increasing the departure fee caused airlines to switch to larger jets, reducing the total number of flights in equilibrium. However, the wealth transfer from the airlines to the airports also decreased the airlines total profits in comparison to the base case. Airports should thus be

Table 12: Theoretical Market Simulation Results (Experiment 2: Departure Fees)

Market	Fee Amount	# of Eqbm.	Largest # of Daily Flights	Largest Total Profit
A (Small)	$1,000	5	16	$54,955
	$5,000	4	12	$37,086
	$10,000	2	8	$17,086
	$20,000	0	0	$0
B (Medium)	$10,000	5	12	$23,024
	$20,000	0	0	$0
C (Large)	$10,000	1	22	$52,910
	$20,000	0	0	$0

wary in instituting departure fees in an effort to reduce congestion, as airlines who see their profit margin disappearing may simply decide to leave for another airport. This outcome occurs in all three markets when the departure fee is increased to $20,000 per flight. Airlines would lose money if they entered the market, so they simply decide to operate no flights.

Experiment 3. Slot controls

Some highly congested airports use slot controls to reduce congestion. In this system, airlines are allowed to bid on “slots” that allow the airlines to the use airport’s runways for a takeoff or landing. Slots are often available in pairs, allowing airlines to operate both an incoming and outgoing flight from the airport. Many airports in Europe use a slot control system, as well as some of the most heavily congested airports in the United States, including New York’s LaGuardia National Airport (LGA), Chicago’s O’Hare International Airport (ORD), and Washington, D.C.’ s Ronald Reagan National Airport (DCA).

For Experiment 3, a slot control of 10 departures is placed on both airlines. This means that no airline can choose more than 10 flights per day on any aircraft. As in the other simulations, this experiment presumes that all aircraft are waiting for takeoff at the departure airport–that is, airlines do not need to spend their slots on landing aircraft from other cities. Table 13 presents the equilibria and total profit that resulted from Experiment 3.

Table 13: Theoretical Market Simulation Results (Experiment 3: Slot Controls (10 Slots))

Market	K_1	F_1	π_1	K_2	F_2	π_2
A (Small)	LNB	4	$48,470	RJ	4	$13,826
	SNB	5	$41,006	RJ	5	$17,283
	RJ	8	$27,652	RJ	8	$27,652
	RJ	5	$17,283	SNB	5	$41,006
	RJ	4	$13,826	LNB	4	$48,470
B (Medium)	LNB	6	$91,020	SNB	6	$51,030
	SNB	6	$51,030	LNB	6	$91,020
C (Large)	LNB	9	$43,290	LNB	9	$43,290

K_i = Aircraft choice for airline i
F_i = Flight frequency for airline i
π_i = Profit for airline i

Since the majority of the equilibria in the hypothetical markets operated fewer than 10 flights per airline, imposing a slot control of 10 departures did not affect the equilibria in the small and medium markets. The number of flights operated in the large market reduced from a total of 22 to a total of 18. In the real world, this would result in flights that are more full, increasing the load factor (the percentage of filled seats on any flight). However, since the model takes the load factor as exogenous, imposing a slot control in these hypothetical markets would lead to a shortage–consumers who are willing to pay to fly but are unable to find a seat. This is not surprising, since microeconomic theory suggests that shortages arise when quantity restrictions are impose.

Discussion of Theoretical Results

The results of the theoretical model appear to confirm my hypothesis that regional jets cause airlines to choose smaller aircraft and more frequent departures. Even though they are more expensive on a CASM basis, regional jets allow airlines to maximize profit by limiting the number of wasted, empty seats. However, airlines' preference for regional jets can lead to congestion, as more flights are necessary to service the same number of consumers.

The policy experiments of removing regional jets from fleets, instituting departure taxes, and implementing a slot control had moderate success in reducing the number of combined daily flights across the equilibria. However, as with most regulation, the reduction in congestion is accompanied by a loss of producer surplus, as airlines are restricted from making the most cost-effective allocation of their fleet. Consumers may be hurt by these types of regulation as well, as frequent flights allow for a reduction in delays and minimize waiting times (Brueckner and Pai 2009). In the case of slot controls, shortages would certainly lead to a loss in producer and consumer surplus.

This creates a dilemma for transportation policy-makers. Congestion is expensive, and the delays caused by crowded airports waste fuel, hurt the environment, and cause tremendous deadweight loss and reductions in consumer and producer surplus. However, regulations of the type modeled in Experiments 1-3 reduce congestion at the expense of producer and consumer surplus. It is up to the policy-maker to weigh the costs and benefits of congestion and regulation to determine what level of corrective action is necessary to maximize total surplus in the air travel market.

5 Results: Empirical Models

Using 2010 supply and demand data from the Bureau of Transportation Statistics, we can use this simulation model to attempt to mimic real-world airline decision-making. Comparing the real-life market outcomes to the simulated results is a good way of checking the predictive validity of the model. Additionally, if the simulator can correctly predict the market outcome, policy experiments can be applied to test congestion-reducing measures in crowded markets.

The simulation model works best when run using two airlines. Running the model with three or more airlines can often result in hundreds of equilibria and take a long time to simulate. The predictive value of these outcomes with many equilibria is not very high. Limiting the simulation to duopolistic markets leads to more useful results. However, this restriction also affects the types of markets we can simulate, as most significant city-pair markets have three or more competing airlines. Therefore, I chose three smaller markets to check the validity of the model.

Table 14: Market Parameters for Empirical Model

Airport 1	Airport 2	Demand	Load Factor	Avg. Fare	Distance	Airlines
PDX	SEA	453994	0.731	$108.49	129	Horizon, SkyWest
BOS	ATL	686493	0.86	$191.75	946	AirTran, Delta
LGA	DCA	416586	0.48	$178.06	214	Shuttle America, US Airways

Market 1. Portland, Oregon (PDX) → Seattle, Washington (SEA)

Portland International Airport (PDX) is a mid-sized airport in Portland, Oregon. It is a hub for Seattle-based Alaska Airlines and its subsidiary regional carrier, Horizon Air (QX). One of the most popular services out of Portland is the short flight to Seattle-Tacoma International Airport (SEA), just 129 miles away. Horizon Air competes on the PDX-SEA route with regional carrier SkyWest Airlines (OO), which flies for United Airlines under the "United Express" brand. Table 15 below summarizes the types of aircraft available for each of these airlines. Table 16 summarizes the simulation results from this empirical model and provides the actual average flight frequency and aircraft size decisions in the PDX-SEA market in 2010.

Table 15: Aircraft Available in the PDX → SEA Market

Horizon Air (QX)	SkyWest Airlines (OO)
de Havilland Dash-8 (RJ)	Embraer Brasilia EMB-120 (RJ)
Canadair CRJ-700 (RJ)	Canadair CRJ-200 (RJ)
	Canadair CRJ-700 (RJ)
	Canadair CRJ-900 (RJ)

Table 16: Simulation Results for PDX → SEA market

K_1	F_1	π_1	K_2	F_2	π_2
Dash-8	16	\$78,867	EMB-120	16	\$24,878
CRJ-700	17	\$78,409	EMB-120	17	\$26,432
CRJ-700	14	\$64,572	CRJ-100	14	\$46,845
CRJ-700	12	\$55,347	CRJ-700	12	\$60,440
Actual Market Outcome (2010)					
Dash-8 / CRJ-700	20		EMB-120	9	

K_i = Aircraft choice for airline i, F_i = Flight frequency for airline i, π_i = Profit for airline i

The results show one of the complications in using this model to predict real-life results. Real airlines are not limited to choosing only one type of aircraft when making their fleet allocation decisions. Indeed, airlines often mix and match different types of aircraft in a given market. For instance, in the PDX-SEA market, Horizon Air used a mix of Dash-8 and CRJ-700 regional jets in 2010, at an average frequency that is slightly higher than the model prediction. SkyWest's choice of aircraft was predicted correctly by the model in two cases, but the model predicted a higher daily frequency than was actually the case.

Market power may be playing a role in these outcomes. Horizon Air is a subsidiary of Alaska Airlines, which maintains a hub at both PDX and SEA. This may reduce costs when flying between these two airports, explaining why the model predicted a lower flight frequency than actually occurred. Additionally, passengers in this region of the United States may have a preference for Alaska Airlines/Horizon Air flights if they can accrue frequent flyer points. This type of brand preference could explain why United, which has a smaller presence in the Northwestern United States, performs fewer daily flights than the model predicts.

Market 2. Boston, Massachusetts (BOS) → Atlanta, Georgia (ATL)

Boston's Logan International Airport (BOS) is a major airport in the northeastern United States. It is a focus city for JetBlue Airways, Delta Air Lines, and United Airlines, and is served by nearly every major U.S. airline. Atlanta Hartsfield-Jackson International Airport (ATL) served more passengers and handled more flights than any other airport in the world in 2010. It is a hub for Delta Air Lines, which is currently the largest airline in the world by fleet size.

The BOS-ATL market is served by two airlines: Delta Air Lines and AirTran Airways. Delta is a major world airline with over 700 aircraft in its fleet. AirTran Airways is a smaller, low-cost airline which also has a hub in Atlanta. It flies two planes: the Boeing 717 and Boeing 737, both of which are classified as small narrowbodies. The tables below show the aircraft available for each airline and the results.

Table 17: Aircraft Available in the BOS → ATL Market

AirTran Airways (FL)	Delta Air Lines (DA)	Delta Air Lines (Continued)
Boeing 717-200 (SNB)	Boeing 737-700 (SNB)	McDonnell-Douglas DC-9 (SNB)
Boeing 737-700 (SNB)	Boeing 737-800 (LNB)	McDonnell-Douglas MD-88 (SNB)
	Boeing 757-200 (LNB)	Airbus A320-200 (SNB)
	Boeing 767-300ER (WB)	Airbus A330-200 (WB)
	Boeing 767-400 (WB)	Airbus A330-300 (WB)
	Boeing 777-200ER (WB)	Airbus A319 (SNB)
	Boeing 747-400 (WB)	

Table 18: Simulation Results for BOS → ATL market

K_1	F_1	π_1	K_2	F_2	π_2
Boeing 717	9	\$91,961	McDonnell-Douglas DC-9	9	\$57,808
Boeing 737	8	\$118,526	McDonnell-Douglas DC-9	8	\$51,385
Boeing 717	7	\$71,525	Boeing 757-200	7	\$77,492
Boeing 737	4	\$59,263	Boeing 747-400	4	\$155,132
Actual Market Outcome (2010)					
Boeing 717/737	4.5		Multiple Aircraft	10	

K_i = Aircraft choice for airline i, F_i = Flight frequency for airline i, π_i = Profit for airline i

Table 18 shows that in 2010, AirTran Airways operated three to four daily flights on average using Boeing 717 aircraft and one to two flights using Boeing 737 aircraft from Boston to Atlanta. Delta Air Lines operated mostly Boeing 737-800, Boeing 757-200, and McDonnell-Douglas MD-88 aircraft in the market.

This market again shows some of the shortcomings of the computational model. Since airlines are only allowed to select one airline type in the simulation, the model fails to capture airlines' ability to mix and match different types of aircraft.

Another key consideration is the presence of connecting passengers. Atlanta is a large hub for Delta Air Lines, and often passengers must first fly to Atlanta in order to connect to another city. This means that passengers may be forced to choose Delta to fly to Atlanta if they wish to continue their itinerary to another airport. The addition of a hub criterion that weights the daily demand towards the hubbed airline would be a potential future modification to the model. Alternatively, data that could isolate passengers who have Atlanta as their final destination versus passengers connecting through Atlanta could also improve the model's predictive capacity.

Market 3. New York, New York (LGA) → Washington, DC (DCA)

New York's LaGuardia National Airport (LGA) and Washington, D.C.'s Ronald Reagan National Airport (DCA) both share similar characteristics. Both airports are located closer to their cities' business zones than larger international airports (John F. Kennedy International Airport (JFK) in New York and Dulles International Airport (IAD) in Virginia). This proximity to centers of commerce makes LGA and DCA favorable for business travelers, who are willing to pay a premium to land closer to the city center. We also see a lower load factor in this market than similar-sized markets, suggesting that business travelers are also willing to pay a premium for the luxury of choosing between many departure times. However, both of these airports are also slot controlled, making airline entry into this market more challenging.

Two airlines compete in the LGA-DCA market. US Airways (US) flies its well-known US Airways Shuttle route between the two cities. The Shuttle service connects Boston's Logan Airport, LaGuardia Airport, and Reagan National Airport on a near hourly basis. Regional carrier Shuttle America (S5) also operates in the LGA-DCA market, flying under the "Delta Shuttle" brand for Delta Air Lines. Like many regional carriers, Shuttle America has only one type of airline in its fleet: the 76-seater Embraer E175. On the other hand, US Airways has a full spectrum of nine different aircraft to choose from, spanning all four families of planes.

The tables on the next page summarize the available aircraft for each airline, as well as the simulation results.

Table 19: Aircraft Available in the LGA → DCA Market

Shuttle America (S5)	US Airways (US)
Embraer E175 (RJ)	Boeing 737-400 (SNB)
	Boeing 737-300 (SNB)
	Boeing 757-200 (LNB)
	Boeing 767-300 (WB)
	Embraer E-190 (RJ)
	Airbus A320 (SNB)
	Airbus A330 (WB)
	Airbus A319 (SNB)
	Airbus A321 (LNB)

Table 20: Simulation Results for LGA → DCA market

K_1	F_1	π_1	K_2	F_2	π_2
Embraer E-175	11	\$55,100	Boeing 737-400	11	\$110,637
Embraer E-175	14	\$70,127	Embraer E-190	14	\$86,219
Embraer E-175	12	\$60,109	Airbus A319	12	\$98,518
Actual Market Outcome (2010)					
Embraer E-175	11.5		Airbus A319	12	

K_i = Aircraft choice for airline i
F_i = Flight frequency for airline i
π_i = Profit for airline i

The model was much more successful in predicting the actual market outcome in this market. In 2010, Shuttle America operated approximately 11.5 daily flights on average from LGA to DCA using their Embraer E-175s, and US Airways used the Airbus A319 to operate 12 daily flights on average between the two airports. This almost exactly matches the third equilibrium predicted by the model.

Discussion of Empirical Results

The simulation model acheived mixed levels of success when applied to empirical markets. While the model was able to accurately predict the real-world outcome in one of the three markets tested, the simulation was off the mark on the other two markets. Additionally, the fact that the simulation could only be used in duopolistic markets with exactly two airlines limits its usefulness.

The model's struggles to predict actual outcomes are likely tied to some of the inherent assumptions in the simulation. For instance, the simulation treats each market in the vacuum–airlines make their scheduling decision without considering impacts in other markets. However, real airlines cannot change their fleet sizes in the short run. Therefore, short-run decision making is constrained by the number and types of aircraft available in the current fleet–these aircraft are allocated over *all* available markets. This behavior is not captured in the simulation.

Several of the markets also showed the shortcomings of allowing simulated airlines to choose only one type of aircraft to operate flights. In the real world, airlines often mix and match the types of aircraft that operate a given route on a given day. For legacy carriers, this behavior may include mixing aircraft from the mainline fleet with regional jets from one or more regional carriers. The assumption that each airline will only choose one type of aircraft in the simulation is therefore unrealistic.

However, the fact that the simulation was able to correctly predict the scheduling outcome in one of the markets shows the model's potential for accurate predictions. Expanding the model to allow for multiple airlines and multiple aircraft choices, as well as considering the impacts of market power and connecting passengers, would increase the simulation's complexity, but perhaps also lead to more accurate results.

6 Conclusion and Future Research

Consumer and airline preferences for regional jets have clearly changed the scheduling landscape in the U.S. air travel market. While regional jets are more expensive on a seat-mile basis, their flexibility and comparatively lower fixed costs allow airlines to make more flexible scheduling decisions. The results of this paper showed that the availability of regional jets do increase the flight frequency and reduce the average aircraft size in air travel markets, *ceteris paribus*. Additionally, while congestion-reducing policies do succeed in decreasing the average number of daily flights, they do so at the expense of producer and consumer surplus. Further research needs to be done to examine whether the benefits of reducing airport congestion outweigh the costs.

Furthermore, the model presented here makes several assumptions that could be relaxed in future research. Particularly, this simulation model worked best in duopolistic markets with exactly two airlines competing. Changing the structure of the strategic game or the constraints could allow for the simulation to be applied to markets with more than two airlines. This would allow the simulation to be used to model a broader range of air travel markets. Additionally, further projects may wish to consider allowing airlines to select more than one type of aircraft in the simulation model. Further verification testing needs to be done using empirical market data to ensure that the model is indeed making accurate calculations of the flight frequency and aircraft size outcomes.

Finally, more research into the implications of the rising popularity of regional jets should be undertaken. Regional jets are profoundly affecting the ways in which airlines make decisions. There is a surprising lack of literature researching the relationships between airlines and their regional carriers, which may not operate using the industrial organization frameworks of decades past. A closer understanding of these markets will allow aviation policy-makers to make more effective decisions about congestion-reducing policies while improving the efficiency of the national air travel network.

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