Combat Evolved: Lanchester's Laws in Modern Warfare

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Abstract

In 1916, Frederick Lanchester published *Aircraft in Warfare: The Dawn of the Fouth Age*, a book which was criticized by many who saw the airplane as having little influence on the battleground. His work was based heavily on a series of differential equations with no support of computer models or simulations, but has since been affirmed by mathematicians and military theorists alike. History now shows that the airplane revolutionized the modern world and brought warfare to advance from a second-generation to a third-generation caliber, where speed and surprise are necessary to military victories.

However, we now move to the 21st century—an age of fourth generation warfare (4GW) where the battlefield is an urban environment. 4GW is characterized to be complex, involving more inputs and influences to the battlefield than the previous three generations of combat science, including terrorism and the presence of civilians. In this age of asymmetric warfare, do Lanchester's Laws still hold true, or have they now become a defunct area of operations research?

We shall see how Lanchester's Linear and Square Laws can still be used today for military forces and their support units. Inspired by the works of Clausewitz and Sun Tzu, we will explore classical war tactics and their applications in today's combat theater.

Dedicated to the Warriors of the Past, Present and Future, especially those born from the Hoya Battalion, and the cadre that mentor the next generation of soldiers.

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Generally, an Artful Strategy must be supported with
A thousand swift four-horse vehicles,
A thousand armored four-horse vehicles,
A hundred thousand armored troops,
And provisions transported for a thousand miles.

Sun Tzu, *The Art of Strategy,* written sometime between 480-221 B.C. [W1]

Introduction: The Math of Conflict

The face of war has changed over centuries, yet the basic strategies and objectives have changed little. In the above passage, Sun Tzu writes from over 2200 years ago about the logistics of moving an army to execute an operation. To compare these words of wisdom to a modern battle plan shows many similarities, but we may even add more to Tzu's work.

Tzu mentions four core elements of the tactical force: light vehicles, heavy armor, infantry, and combat support. Each element has a quantitative value assigned to it, and this is similar to a coefficient in a polynomial equation. Is this a coincidence? Surely not, yet as it is with other sciences, mathematics is able to express and relate multiple circumstances in an understandable fashion to form models of real life scenarios that are reliable. Military science is no exception, and its value has ranged from aiding a swift victory, to finding prisoners of war, to minimizing casualties and collateral damage.

The four elements above are still relevant in modern warfare, although they are vastly more advanced and equipped with more firepower and stamina. Let us examine these "core four" and their quantitative values:

- Light vehicles (*"a thousand swift four-horse vehicles"*) often referred to the chariot ranks featuring two-wheeled chariots led by a team of four horses in a single row. Crewed by a driver and an archer, some chariots would also feature a soldier armed with a spear. Often defeated by infantry in battle [M1], these chariots were the predecessors of today's light vehicles in the military arsenal, most notably Humvees [USA 1]. These units are agile, fast, and less protected and rely on their speed to outlast opposing forces.

- Heavy armor (*"a thousand armored four-horse vehicles"*) speaks of military units that are able to be effective in a direct line of fire. As with the chariots mentioned before, improvements made to better defend the armored chariots would include a car constructed from metal, thus weighing more and increasing the chances of being bogged down [M 2]. However, their armor proved effective in defending onboard archers, turning a chariot into a firing platform with some amount of versatility. In the 21st century, armored vehicles still serve the same purpose and are made to withstand brutal hits from enemy fire. Their added weight hinders their speed and agility, and with modern technology, it does not take a tank to eliminate another tank from battle, but rather, shoulder-mounted weapons manned by personnel are capable of rendering a tank of armored personnel carrier useless [USA 2].

In this brief discussion of the *four-horse vehicles*, we have noted their differences and how both still live on in the 21st century battleground. We must also observe Tzu's reasoning of demanding that both of these forces are requested in equal numbers: a thousand each for the offensive strategy. Assuming by a quantitative analysis that the equal numbers imply that neither type of unit is more important than the other, Tzu has shown that both light and heavy vehicles are necessary and vital to a battle plan. That is, we cannot do without them, yet no one mechanical marvel is superior to another, for both have weaknesses and strengths. In trading off some speed, a chariot can gain armor, or on the contrary, a chariot may trade off some armor to be more agile on the battleground [M 3].

- Infantry (*"A hundred thousand armored troops"*) remains to be the sole unit of warfare that has changed the least in its role over centuries of conflict. The infantry have the versatility of maneuvering across any terrain and forming multiple combinations of groups. The basis for an infantry soldier remains to be an individual wielding his own weapon and being able to be independent from other infantry soldiers, though in most cases, the combined force power of an infantry unit is greater than the sum of its independent soldiers' force powers. In modern warfare, the infantry is still the centerpiece of any conflict as described by Tzu, and reinforced by Machiavelli's writings [M 3].

- Combat Support (*“provisions transported for a thousand miles”*) has grown over centuries of warfare from the time Tzu penned *The Art of Strategy*. In Tzu's time, combat provisions would include mechanical support for chariots, food and water for personnel, advisors, and a variety of weapons that were kept as reserves. By modern comparison, the age of firearms and mechanized warfare have added on components of fuel, ammunition, and more spare parts. Perhaps the greatest anomaly from Tzu's writing on the combat provisions though, is the distance that today's armies can travel with their own support. First, let us note that the translation of “a thousand miles” must take into effect that the Chinese equivalent for a mile, li, is literally translated as one league. However, a Chinese league is not the same as the same measurement as used by Christopher Columbus. It is instead, the inverse, whereas roughly five and a half kilometers form a league for Columbus, a traditional Chinese league is about a half kilometer, though in Tzu's time, this varied between 500m and 450m [W 1]. To take a literal comparison, a western translation of “provisions transported for a thousand miles” should read “provisions transported for a half thousand miles” if we are to be legalistic about the numerical value assigned to the logistics arm of Tzu's strategy. Any analogy now to bridge between Tzu's war game and the modern era shows that a half thousand miles of combat support is only a fraction of that needed for modern combat. Consider the Pacific Theater of World War II, which is agreed to have begun at Pearl Harbor, Hawaii, and concluded at Hiroshima and Nagasaki, Japan. The range between the island of O'ahu, home to Pearl Harbor, and the island of Honshū is roughly 4,242 miles—a staggering figure that dwarfs the half thousand miles Tzu describes, not to mention that much of the Pacific Theater's war included the dominion of the sea. By any measure, the greatest quantitative difference between Tzu's writing and conflict as we know it today is perhaps this arm of combat support and logistics. Not only has the demand for support increased, but the range over which units depend on support has grown immensely to the point where we are now able to send provisions to any theater in the world.

Figure 1: The value of mobility is proven with expeditionary forces, such as this USMC LAV-25 (in the foreground). In the background sits a hovercraft, capable of deploying from a larger amphibious assault ship over the horizon from the beach. Contrasting Tzu's writing of "provisions transported for a thousand miles, this Marine unit is more than 8,000 miles from its home base in California [M-USAF]

Indeed, there is a relationship between science and conflict—and let us only speak of such in context of its quantitative and absolute parts. That is, without taking into account the politics of war, one is able to conduct rational and decisive research on the basis that each combatant operates to maximize their gain with the least loss. In game theory, this is the fundamental theorem, known as the minimax theory [CN 1], and it is especially useful when analyzing whether a possible solution to a given scenario is possible. However, an insight into the delicate theory of the battlefield strategy shows that the research by a single British mathematician in the early 20th century reveals more to the battlefield than simply an arena of combat.

Frederick William Lanchester, a talented engineer in the early automotive industry, is responsible for early research in aerodynamics and automotive technologies that are still in place today, such as detachable wheels on the cars made at the Lanchester Motor Company. However, Lanchester's most significant contribution to our world may not be the accelerator pedal nor steering wheels in cars, although we do commonly utilize those patents. Instead, a set of mathematical formulae known as *Lanchester's Laws* have shaped history since World War I by means of their application to business models and 20th century warfare [CS 1].

Indeed, Lanchester was neither a mathematics nor military science student at Imperial College, Kensington. Furthermore, his academic record proved to be disappointing and having attended a preparatory school and boarding school, Lanchester found no need to be serving in the military. However, at the outbreak of World War I, Lanchester was in his mid-forties and was an accomplished businessman in the automotive industry. Having commenced studying aeronautics in 1892, Lanchester furthered his research by predicting how the airplane would change the face of modern warfare. Note that at this time, the airplane has not been invented yet, and powered flight would finally be earned by mankind in 1903 when the Wright brothers launched *Wright Flyer I.*

In spite of this, Lanchester was already interested in the mathematics on a battleground prior to the arrival of warplanes. His foresight of the airplane being a war machine drew much scrutiny and criticism, and his most famous writing, *"Aircraft in Warfare: The Dawn of the Fourth Age"* [L] was not well received in the United Kingdom. Nevertheless, the potential was recognized when the United Kingdom armed forces recognized the possibilities of using aircraft in combat, and prior to his hypothesis regarding aerial combat, Lanchester had mathematically researched ground combat.

Since the publishing of *"Aircraft in Warfare",* the face of war has changed and while Lanchester's Laws were useful in the early 20th century for the Allies in combat, this theory of attrition is now used more in logistics and business.

However, Lanchester is credited with being the co-founder of operations research, a field of study working with mathematical modeling and algorithms to seek optimal solutions to extremely complex problems. Operations research is responsible for many military successes in both world wars, from the studies of naval convoy groups to using Lanchester's laws in bombing campaigns against ground targets and submarines. Thus, Lanchester's laws have seen many a time where their usefulness has been a part of shaping history.

Our business here is to answer no simpler a question, than whether Lanchester's laws still apply today in a world of asymmetric warfare and fourth generation warfare. We have seen that in conflicts such as Iraq, that the front line seldom exists, and force sizes are difficult to quantify. Yet, if Lanchester's mathematics held true to battles in the past, why should it not for the fight beyond today?

Lanchester's Linear Law

Lanchester's first law of conflict is based on ancient combat, where upon engagement, one man can only kill another man at the same time due to close combat ranges involving weapons such as spears, swords, and battle-axes, while defensive measures included shields, chain-mail, and footwork. Thus, Lanchester wrote that the definition of this law applies to “unaimed fire” [MN 2], which does not treat the verb “fire” as a term of weaponry, but this refers to the direction of lethal delivery of a weapon. More specifically, the common case for which the linear law applies is for a cannonade, or artillery barrage. It should be made distinctive that this law is not applicable to today's modern era of war where the artillery barrage is a ground support instrument that is far more precise than the cannons of Lanchester's day and past centuries.

By intuition, one can imagine the one-on-one combat as two groups with some function between all of their elements. For instance, to demonstrate that three soldiers of group B were killed by a single member of group A, then we designate that member of A as having an effectiveness ratio of 3:1, where for every soldier who can eliminate three opposing soldiers, they are three times as effective. In the same way, this ratio applies to other units, such that each unit is defined as having an effectiveness in comparison to the enemy.

Given these ratios, one can deduce that for any two forces of the same size, the one with the higher fighting effectiveness will be the victor, and the other will suffer a total loss. However, we are interested in knowing, for instance, what *size* ratio must a superior fighting force have to be able to overpower an inferior fighting force, using the minimal amount of troops. Conversely, it is also of interest to find the size ratio of our inferior force should we wish to guarantee that they can defeat a better trained fighting force.

However, such a system requires only a system of linear equations to solve for a solution. Suppose R is the number of red troops to begin with, and r is the effectiveness index of the combined red force. Similarly, suppose B is the number of blue troops to begin with, and b is the effectiveness index of the combined blue force. As stated above, the effectiveness ratio is the number of enemy troops that the average soldier on the respective side is expected to kill. Note that while B, R must be integers, b and r can only be any non-negative real numbers. Thus, for our equations:

rR is the total number of blue troops that will be slain by red forces, and

bB is the total number of red troops that will be slain by blue forces.

Then setting rR-bB=k, where k is a real value, we investigate what implications are present if k is negative, positive, and zero. In the trivial case, when k=0, then rR=bB, thus both forces will wipe each other out. Should k>0, then rR>bB, which means that the total number of blue troops slain is greater than the number of red troops slain, thus red is the victor. Similarly, should k<0, then bB>rR, which means that the total number of red troops slain is greater than the remaining blue troops, thus the blue forces claim a victory.

In such a system, our calculations are simple, and provide some sense of direction where we want to explore when we move from this ancient world of warfare to the age of gunpowder and aimed fire. Note also that our system of linear equations above can also be modified to accommodate a shock attack, such as the opening salvo-barrage of artillery which can demolish a first wave of attackers.

Suppose that red barrage a rain of arrows upon blue forces when the first wave of blue forces approaches within an arrow's optimal range. That is, given a mass unleashing of high-trajectory shots, there is at any one time, a maximum of R arrows in the air, if every red soldier fires one arrow. Let us not assume that every arrow shall meet its target, nor that every arrow misses. Therefore, let us set a constant B_1 which represents the first wave of blue troops. Subsequent waves will be denoted B_2, B_3 and so on, and let (B_1 - a_1) represent the number of troops in the first wave that were not incapacitated by the arrows from red forces, where a_1 is the number of arrows in the first wave that were fired and hit blue troops. Then the (B_1 - a_1) troops are able to regroup and perhaps go on to fight the red troops.

Calculating the same for each wave, and assuming that red has suffered no losses yet, we modify the above equation:

$$rR\text{-}b[(B_1 - a_1) + (B_2 - a_2) + ... + (B_{n\,-}\, a_n)] = k$$

where n is the total number of waves that blue is willing to send. Should blue forces arrive at such a point that no wave encounters arrows, then the a_n value can be set to zero, and the above equation is solved for *k*. Similarly, adjustments can be made for red forces too, and by linear addition laws, the above equation can be rewritten:

$$rR - b[(B_1 + B_2 + ... + B_n)\text{-}(a_1 + a_2 + ... + a_n)] = k$$

and since $B_1 + B_2 + ...+B_n$ is a partition of the blue forces, B, then we can again rewrite this equation:

$$rR - b[B\text{-}(a_1 + a_2 + ... + a_n)] = k$$

Finally, distributing this gives $rR - bB - b(a_1 + a_2 + ... + a_n) = k$

Clearly, since the *a* terms are all positive, this equation will determine that $k>0$ for a red victory, which makes sense since blue has already lost many of its troops to the onslaught of arrows.

Linear systems can be interesting to play around with, but they are limited in a field of warfare defined by close quarter combat, and require little need for differentiation to analyze. Therefore, Lanchester went about finding another important law that would be applicable to the age of warfare where forces could engage further apart with firearms.

Lanchester's Square Law

The sequel to Lanchester's Linear Law is the famous *N^2 Law*, most commonly used as a comparison between two forces by their numbers. Simply put, the N^2 Law states that the power of a force is proportional to the square of the number of units for forces engaging with aimed fire. This is differentiated from the previous, linear law, which applied to unaimed fire weapons for all combatants. Before moving on, we should also clarify that unaimed weapons can be used for offense and defense. However, with the Square Law, we cannot include defensive measures as weapons since all defensive weapons ultimately have a weakness in the aimed fire model [MN 1]. We shall see how this later plays out in the offensive versus defensive discussion, based on Clausewitz's works [CC 2].

The aimed fire model begins with setting up two functions, *R(t)* and *B(t)* where each represent the number of troops fighting at any time t>0. The original size of each force is *R(0)* and *B(0)*. Then suppose, as with the above linear law, that each unit in red force can destroy *b* blue force units, and similarly, each unit in blue force can destroy *r* red force units.

Thus the rate of red forces being lost is $\frac{dR}{dt}=-bB$ and the rate of blue forces being lost is written as $\frac{dB}{dt}=-rR$. These differential equations depend on the explicit variable *t*, so we shall divide the second by the first to obtain:

$$\frac{(\frac{dB}{dt})}{(\frac{dR}{dt})}=\frac{dB}{dR}=\frac{-rR}{-bB}=\frac{rR}{bB}\rightarrow\frac{dB}{dR}=\frac{rR}{bB} \quad .$$

Then cross-multiplying: $bB\,dB=rR\,dR$.

Integrating both sides gives $\int bB\,dB=\int rR\,dR$ $\rightarrow$ $bB^2 = rR^2 + k$, where *k* is a constant

Thus, $bB^2 - rR^2 = k$.

This *k*-value can show which which force wins depending on whether it is positive, negative or zero. The trivial case, when k=0 occurs when $bB^2 = rR^2$, that is, when both forces have an equal combined ratio of force sizes and effectiveness. In this case, both forces are eliminated through battle—the only solution for either force to win would be to not engage in battle.

Now putting the trivial case aside, suppose k>0, so then this means that $bB^2 > rR^2$, meaning that blue holds a victory. Conversely, if $k<0 \rightarrow bB^2 < rR^2$, red owns a victory.

We now turn to use these differential equations to prove a well-known element of war theory.

Division of Forces

"The other method of turning the enemy, and cutting off his retreat by dividing our force, entails the risk of attending a division of our own force, whilst the enemy, having the advantage of interior lines, retains his forces united and therefore has the power of acting with superior numbers against one of our divisions."

General Carl von Clausewitz, *Vom Krieg (On War)*, early 19th century [CC 1]

A classic military maxim is to almost never divide your forces to engage the enemy [MN1], as detailed above in Clausewitz's famous writing, *On War*. Simply put, every division of force power significantly weakens the sum of the forces. We show this by using the Lanchester's square law, where given R as the total of red units ($R=r_1+....+r_n$), we see that $\frac{1}{R^2}$ is going to be larger than

$\sum(\frac{1}{r1^2})+...+(\frac{1}{rn^2})$ for all $r \in \mathbb{N}$.

For example, when R=2, $\frac{1}{2}=\frac{1}{4}+\frac{1}{4}\rightarrow(\frac{1}{2})^2=\frac{1}{4}>(\frac{1}{4})^2+(\frac{1}{4})^2=\frac{1}{16}+\frac{1}{16}=\frac{1}{8}$

Thus, assuming Lanchester's power law holds true for all types and quantities of military units, the force power of a unified force is always greater than the sum of force powers of that force divided.

Suppose we neglect all other factors and concentrate upon the quantity and quality of two fighting forces, red and blue [MN 1]. Given that red is an inferior force, blue units are defined to be three times as effective as red units, thus b=3r. However, red is able to wield a force with twice the numbers of blue force, so $R_0=2B_0$. Then in the course of one battle:

$$rR^2-bB^2 = r(2B_0)^2 - 3rB_0^2 = 4rB_0^2 - 3rB_0^2 = rB_0^2 > 0$$

Thus, this shows that red force, having an inferior but larger force, is able to defeat blue force while both forces are still unified. However, suppose that the opportunity arises for blue units to divide red forces into two groups, perhaps using terrain features to their advantage. Having not detailed *how* the red forces are less effective, we can assume that blue units may be more intelligent, or are more mobile.

We set this up so that blue forces have already divided the red forces, so that in the first round of engagement, the same number of blue units will face red units. Intuitively, blue units are more effective, so we expect a win. Let us show this via battle I:

I: B_0 blue vs. R_0 reds

$r(B_0)^2 - 3rB_0^2 = -3rB_1^2$

where B1 is the leftover blue forces who were not eliminated in battle I

Then solving for the left-hand side: $r(B_0)^2 - 3rB_0^2 = -2rB_0^2$

Thus $-2rB_0^2 = -3rB_1^2 \rightarrow B_1^2 = (2/3)B_0^2$

So then $B_1 = \sqrt{(2/3)}\ B_0$

This means that after the blue force has wiped out the red force in the first battle, about $\sqrt{(2/3)} \approx 82\%$ of the original blue units are still available to fight. These blue units move swiftly to engage with the second group of red units that were separated from the first group that has just been eliminated. Therefore, our setup for battle II substitutes $B_1 = \sqrt{(2/3)}\ B_0$ for the new blue force quantity, and this time, red has more troops to begin with than blue does since blue only has 82% of its original units, B_0, but this section of red units is still at the same quantity as the original number of blue units ($R_0=B_0$).

II: $\sqrt{(2/3)}\ B_0$ blue vs. R_0 reds

$r(B_0)^2 - 3rB_1^2 = r(B_0)^2 - 3r(\sqrt{(2/3)}\ B_0)^2 = -rB_0^2 < 0$

So blue has won with $3rB_2^2 = rB_0^2$ and we solve this for B_2: $B_2 = \frac{1}{\sqrt{(3)}} B_0$

Thus, blue forces, having split the red forces into two sizes equivalent to blue's original force size, is able to overcome the 2:1 outnumbering of its troops. From B_2, we find that approximately 57% of the original blue force has survived the two battles. Thus, red forces have been eliminated and blue has suffered a 43% loss. Further investigation shows that if the remaining blue forces stumble upon another red force of the original blue force size (B_0) then the result would be a stalemate: all units on both sides would be wiped out and leave no survivors.

Now suppose that the N-battle sequence is recalculated for a generalized range of effectiveness of the blue troops. That is, suppose as blue force, that we are able to assess our effectiveness as a ratio to the enemy's effectiveness in battle. This effectiveness index is derived from Lanchester's $b=\beta r$ where β is any positive real number, such that we write $\beta=\frac{b}{r}$. Since a β=0 would mean that the *b* is zero, that is realistically impossible to have red forces fight a blue force of zero effectiveness, and β cannot be negative since both *b* and *r* are positive real numbers. However, β need not be an integer.

To demonstrate how powerful the β index is in determining a battle's results, let us expand our combat theater for the red forces, giving the red forces five-fold blue forces' quantity. That is, for every blue unit on the ground, there are five red units available for combat. Analogous to the previous situation, suppose that blue forces are able to partition red forces into five separate groups such that each group has the same number of troops as blue's entire force.

Now, blue will sequentially engage each group of red units. Thus, 100% of blue force engages with 20% of the overall red force; that is, 20% of all red forces is the same size as the entire blue force. With different β indexes, we can assess how each N-battle sequence ends for the blue and red forces, by solving Lanchester's Aimed Fire Model and substituting for the blue troop quantity each round.

<u>N=1, β=1:</u> $rB_0^2-rB_0^2=0$ shows that all blue forces and the first red group are eliminated, leaving the other 80% of red forces intact.

N=1, β=2: $rB_0^2 - 2rB_0^2 = 0$ shows that blue forces have won the first battle, and the leftover blue troops are solved for with $B_1 = \frac{1}{\sqrt{(2)}} B_0$; so about 71% of the original blue force survives for the next round.

We continue with these calculations, with the data expressed in the below table detailing a rounded percentage of the original blue force that has survived each battle. Having completed the first column for N=1, we find all the B_1's for each value of β and can generalize it:

For $R_0 = B_0$ *and* $\beta \in \mathbb{N}$ *then* $B_1 = \sqrt{(\frac{(\beta-1)}{\beta})}\ B_0$, *thus* $\sqrt{(\frac{(\beta-1)}{\beta})}$ *% of* B_0 *survive the first battle.*

A similar calculation moves through the battles sequentially and we stop when blue forces are depleted. The result is a table as shown below, where blue shaded cells represent a blue victory, and red shaded cells show a red victory. White cells are "stalemate" results where blue force and the group it engaged were both wiped out.

Table 1:

		N (battle sequence)				
		1	2	3	4	5
β Effectiveness	1	0%	80%			
	2	71%	0%	60%		
	3	82%	58%	0%	40%	
	4	87%	71%	50%	0%	20%
	5	89%	77%	63%	45%	0%

The results from Table 1, generate enough information for us to generalize further than our B_1 case. A simple pattern occurs with each iteration on the table for increasing N—remembering that the number of troops remaining after each battle is recursively defined as the number of troops that preceded it. Thus, some algebra shows that $B_n = \sqrt{(\frac{(\beta-n)}{\beta})}\, B_0 = \sqrt{(1-(\frac{n}{\beta}))}\, B_0$ (1)

We will refer to this as formula (1), and call it a generalization because while mathematically it is sound, cases in the real world demand that this is not perfectly accurate, although its simulation of casualties and attrition warfare are close to data gathered from Nelson's tactics at Trafalgar, the Battles of Iwo Jima, Ardennes, and Kursk [MN 2]. However, numerous papers debate the verity of using Lanchester's models for these scenarios due to the disagreeable numbers of casualties in each battle that cannot be accurately determined [SR].

In J.H. Engel's paper, *"A Verification of Lanchester's Law"*, the Battle of Iwo Jima is accounted for with Lanchester's differential equations including reinforcements, which we will discuss later in "Reinforcements and Supply Units" along with a response from Colonel Robert Samz, a professor at the United States Military Academy at West Point.

What the B_n Generalization Means for Warfare

By now, some intuition will have shown that in order to have the best survivability in conflict, an army must either outnumber or outwit its opponents. The B_n generalization from above in (1) justifies this case, since by analysis of $B_n = \sqrt{(1-(\frac{n}{\beta}))}\, B_0$, we observe that keeping the $\frac{n}{\beta}$ value low is in the interest of sparing casualties. Thus, if $\frac{n}{\beta}$ is to be kept low, we can either increase β or decrease *n.* Let us analyze each of these possibilities, beginning with the latter.

The number *n*, is the number of iterations it will take for both forces to have a fight to the finish, with either both being eliminated or one force remaining with lesser troops. Although mathematically, the question will be, "Can we reduce *n* to only 1?" the battlefield commander wants to divide the inferior enemy forces into *n* components such that $\frac{1}{n}$ part of the enemy's force is less than the size of the superior force. Thus, *n* realistically depends on the ratio of troops on each side, so this throws us back to Lanchester's law, where in order for the outnumbered force to win, it must be able to divide the enemy, and splitting enemy forces is not an easy task. In fact, the battlefield commander may even need to expend more resources in dividing the enemy, than in fighting him. Therefore, reducing *n* is dependent on an incredible amount of factors, including the terrain that may or may not be capable of holding *n* groups of the enemy while keeping enough distance between each to make them distinctive.

On the other hand, it is a lot easier for an army to control the β index if it follow's Sun Tzu's well-known maxim: "If you know the enemy and know yourself, you need not fear the result of a hundred battles. If you know yourself but not the enemy, for every victory gained you will also suffer a defeat." This fact is a necessity to increasing the β index since it is a ratio of an army's effectiveness to its opponent's. However, should one not be able to estimate the enemy's force effectiveness, an army can always improve its own force effectiveness by upgrading training, equipment, protection, capabilities, and intelligence.

In the November 2008 issue of *Army*, Dennis Steele's article, *"Basic Training Remix"* [SD], gives an insight into how the United States Army is improving its force effectiveness by enhancing basic training, or boot camp. The new "outcomes-based training" shifts the focus of training each soldier to perform their assigned tasks, with some basic combat knowledge for all, to making every soldier a warfighter and lifesaver.

> *"Compared to their pre-9/11 counterparts, they are far more tactically proficient and far more mentally prepared to go to war—they come expecting to go to war, and they know that what they are learning may save their lives," Col. Currey said. He said that outcomes-based training is about higher standards and effectiveness and getting away from a "checklist process."* [SD]

This method of improving force effectiveness can be quantitatively measured by comparing data across the different generations of soldiers. As of the 1st of October, 2008, Basic Combat Training at Fort Jackson, S.C., is now ten weeks long instead of the previous eight week program, and other army training centers around the United States will soon follow suit. Above all, BCT now emphasizes a core factor of making a soldier a warrior: marksmanship.

While previously, soldiers were required to only shoot at targets at the firing range, today's recruits undergo a rigorous program of weapons handling, beginning with receiving their rifle within three days of arrival at BCT. Additionally, basic marksmanship skills are reinforced, such as zeroing, grouping shots, and some theory on ballistics and trajectory flight. Perhaps the most beneficial change is that recruits now undergo a series of urban environment shooting, discriminating between multiple targets to avoid friendly fire, and shooting under stress in situations such as convoy ambushes [SD].

On the training platform, it is obvious that such drills aim to hone a soldier's skill. However, the true beauty of the new Basic Combat Training is that the force effectiveness improvement can be quantitatively examined and justified. With today's technology, an automated firing range gives real time information about live-fire accuracy and the Engagement Skills Trainer (EST) at Ft. Benning, GA, is an indoor combat simulator designed to precede the live fire course [TD]. This digital system has the capability to calculate, store, and analyze data regarding a soldier's precision with a simulated rifle. The feedback includes information on how much recoil the shooter allowed, their trigger-holding technique, and reaction time. For tank crews, a simulator that mimics an M1A1 Abrams tank can similarly quantify data that helps approximate a force effectiveness index [TD].

Fig. 2, Left: At Ft. Benning, GA, recruits use the Engagement Skills Trainer to hone their marksmanship skills prior to using a live rifle [TD]. Fig. 3, Right: To help tank crews get acquainted with their new rides, the M1A1 tank simulator looks and feels the same as the US Army's Main Battle Tank (MBT) on the inside. Training modules such as these simulators allow a force to increase its fighting effectiveness index, without needing to know the opponent's training program and effectiveness [TD].

Therefore, independent of the enemy's effectiveness index, the β index can be increased by one fighting force by making improvements to itself, whether through training or technology. Having done so, it can guarantee that its fighting effectiveness is greater than the enemy's, thus the β-index is greater than one, giving the better fighting force an advantage in battles.

Military Intelligence

One more component indirectly adds to the fighting effectiveness of a force—intelligence. Without going into too much depth, let us observe Tzu's writings again [W2], from *The Art of War:*

"In the Work of the Entire Force,

Nothing should be as favorably regarded as Intelligence;

Nothing should be as generously rewarded as Intelligence;

Nothing should be as confidential as the Work of Intelligence"

We may not be able to quantify intelligence or sort its information according to how much each piece is worth. However, an army with information leads an edge over its opponent lacking in knowledge. Better yet, an army that is good at deceiving the enemy gives them false intelligence and moreover, this corruption of information in the enemy's ranks leads to an added element of surprise, which is a key factor of any attacking element.

The importance of intelligence has never been proven greater than the famed missions of Operation Mincemeat, Operation Barclay, and Operation Fortitude. All three operations were conducted during World War II, as Allied deception plans to draw enemy attention away from the actual Allied objective. In Operation Fortitude, the Allies set up fictional army units and allowed them to be photographed by German spy planes to convince Germans that a massive buildup of Allied forces would land at the Pas de Calais, the closest part of France to England. Nested within Operation Fortitude was Operation Quicksilver, designed to give the Germans the impression that the Allied force was composed of two Army groups. Fueling the bad intelligence, the Allies set about constructing fake buildings, inflatable tanks, and emitting an incredible amount of false radio traffic [ME].

Therefore, there is a difference between the impact that intelligence makes upon the β index compared to the technological and training factors—intelligence work includes counter-intelligence, as one of the Five Uses of Intelligence [W2] that Sun Tzu describes:

"There is Local Intelligence;

There is Inside Intelligence;

There is Counter-intelligence;

There is Deadly Intelligence;

There is Secure Intelligence"

As such, one can directly lower the enemy's effectiveness index by launching counter-intelligence operations, thereby gaining information on the enemy while feeding them deception. Ultimately, it may be a more efficient option to increase the β index by way of intelligence operations than anything else [TR]. Furthermore, this reinforces the idea on page 18 that knowing the enemy and knowing of friendly forces is always an advantage, and the deceptive operations listed above prove that with some amount of creativity, the wrong information fed to a superior force can give the less stronger force a chance to succeed at their mission [ME].

Small Operations versus Large Operations

Having discussed how to minimize the $\frac{n}{\beta}$ value, this should have aroused some debate over how legitimate the argument for a guaranteed victory to the stronger force can be. Let us remember that strength is defined by Lanchester's Law as the square of the number of units on each side, and we can also factor in the effectiveness of each of those combatant units. Now suppose there is a small unit operating in terrain where a large force is known to travel through on supply missions, and the small unit sets itself up to ambush the large unit. With the element of surprise on their side, the small unit is outnumbered, and according to Lanchester's square law, the power of each force shows that if we designate the small force as blue, B, and the large force as red, R, then:

If $B < R$, then $B^2 < R^2$ for all $B, R > 0$ since force sizes are always positive integers

Thus, by Lanchester's law, it seems that the small force would lose this engagement, but let us introduce some Predator-Prey theory here. Also based on differential equations that look similar to what Lanchester's attrition laws have, Predator-Prey equations are known as Lotka-Volterra equations [CN 3], and are non-linear, first order equations designed to show how a biological system interacts. If we treat our predators as the smaller, blue force, and the prey as the larger, red force, then in the above situation, the blue force has a target rich environment. That is, the blue force is outnumbered so much with enemy forces that it will have to select which ones must be targeted first. On the contrary perspective, if red forces are able to fight back, it will take a shorter amount of time for red forces to eliminate blue forces, compared to the time it would take for blue forces to destroy red forces.

However, what the latter statements do not account for is the range of effectiveness for each force's members. We have also not defined how much larger the red force was in comparison to the blue force. In the combat theater, this is easily recognizable as a tactic of guerrilla forces in unconventional warfare. The 21st century has already demanded the importance of special forces that engage in unconventional tactics, such as the Navy SEALS, Army Rangers, and Green Berets [MN 3].

Let us then define "small operations" and "large operations" as a comparison of the two combatant forces. Suppose a mission is defined as a "small operation" if the size of the red force is *n* times larger than the blue force, and conversely, a mission is defined as a "large operation" if the size of the red force is less than *n* times larger than the blue force. By using this *n*-ratio, we are avoiding giving quantitative figures to a real world scenario, and generalizing each scenario to a given proportion. For instance, it is a common ground warfare rule to attack an enemy's position only if you are certain that you have three or more-fold the size of their force [DP 2]. Thus, if an enemy's force numbers twenty, a minimum of sixty soldiers in your support should be available to engage. An even greater ratio is required for beach assaults such as the invasion of Normandy—for every one enemy soldier on the beach, you will need five assault troops to guarantee a victory on the beach assault phase.

Such is war, dependent upon the individual soldier, yet also dependent upon the force as a whole. As shown above on page 11, a greater force power is projected when your force size is larger. Perhaps this is a loophole in Lanchester's laws though, because we have seen, in many a recent year, the effectiveness of small operations involving platoon-sized units, taking on the same missions that large operations would. For example, the first elements of Coalition forces that entered Iraq in 2003 were not cruise missiles or attack planes, but special forces troops. Each team usually operates with less than a dozen soldiers, providing enough men for a squad operation, while being few enough to avoid detection. Therefore, this whole concept of small versus large operations is an intense part of operations research, because if a small, elite group of men can perform the same task as a battalion that is inferior in combat effectiveness, then why would a battlefield commander select the greater number of troops to send if it will cost more logistics, transport, and personnel?

Perhaps if we reexamine the $\frac{n}{\beta}$ value, it may help. In order to minimize this, we want a high β index while maintaining the lowest *n* value possible. Thus, in a large operation mission, we have a lower β index since the men are less proficient in their craft, and either the *n* value is kept low because the large unit will be able to overwhelm the enemy, or the *n* value also has the potential to stay high if the large unit attacks their objective in waves. Common sense would not dictate the latter, as above calculations show. On the other hand, the smaller elite unit has a higher β index due to their proficiency and high standard of training. We shall also assume that the target rich environment provides optimal operation circumstances for the elite unit to work in: many targets, and few shooters, thus the chance of cross fire and fratricide is reduced and our elite small force will not be divided since it is already small, thus the *n* value remains to be 1. Should the *n* value not be one, it gives the enemy, a much larger force, the opportunity to pursue a small group of elite soldiers, and in pursuit, the advantage leans towards the larger force of enemy.

Fig 4, Left: First Marine Regiment, Echo Company, participates in Rim of the Pacific (RIMPAC) exercises in 2006, demonstrating support and security roles during a beach assault in Hawaiian waters. A typical Marine company numbers over a hundred personnel, so we recognize this as a large force operation [H-USN]. Fig. 5, Right: Two U.S. Navy SEALs assume a defensive position on the beach following an amphibious assault. These special forces are considered to operate on a small force operation, with platoons numbering 16 men in total. A typical mission may see as few as eight men in action, so the beta-index between the SEALs and their adversaries needs to be extremely high to ensure success [L-USN].

Now consider the two images above: the same scenario is taking place—the objective is to assault and seize a beach and hold it. On the left, we have a regiment of Marines who are each equipped with heavy gear—note the large assault pack and kevlar helmet. On the right, two U.S. Navy SEALs don no armor and travel lightly. Without knowledge of their respective objectives, we must understand that neither force can accomplish every mission. While the SEALs lack the same amount of support that the Marine regiment has, it is a smaller unit, thus able to spring the trap of surprise and encounter larger units with more stealth. On the other hand, the Marine regiment can simultaneously deliver more firepower and spread that over a wider area, while at the same time being more vulnerable to crossfire.

In Niall McKay's paper, it is argued that since the small force will find many targets to pick on in the early stages of engagement, then the rate of losses for the larger force will be high in the early stages of battle, but on both sides, losses will plateau out as the originally larger force is reduced to a size comparable to the blue force. Eventually, McKay argues, it may be a linear battle that is being fought, redirecting us back to Lanchester's linear law [MN 3], and Svend Clausen, in his defense of calculated warfare, proves that the "guerrilla model also is a time and state continuous model in practice often assumed to be a kind of mean value model. That is, given some average value of loss, *m*, the total number of units remaining in the larger red force can be expressed approximately as:

$R(t)=B(t)m+c, c\in\mathbb{Z}$ where *R(t)* represents the total units in the red forces at time *t* and *B(t)* represents the number of guerrilla units able to fight, plus a constant *c* for a given advantage on either side. For example, a negative *c* means that perhaps blue units were able to have the initiative of surprise, thereby immediately killing *c* red units before red forces could return fire.

Obviously, the equation just described is linear, and is a possible model for the guerrilla model.

Reinforcements and Support Units

As aforementioned, J.H. Engel dissects the Battle of Iwo Jima in *"Verification of Lanchester's Laws"* and accounts for the reinforcements. However, we note that the differential equations given in the above section on Lanchester's Square Law do not account a variable for reinforcements or support crews. Thus, as we modified Lanchester's Linear Law, let us do similarly for the Square Law, for red forces representing the United States Marine Corps, and blue forces represent the Japanese forces:

We have that $\frac{dR}{dt}=-bB$ and $\frac{dB}{dt}=-rR$

Suppose that the function *C(t)* is the rate at which American troops arrive as reinforcements at time $t>0$. Then for the differential equation representing the rate of addition/subtraction of American troops,

$\frac{dR}{dt}=C(t)-bB$ and the Japanese rate of loss is $\frac{dB}{dt}=-rR$ since they have no reinforcements.

In order for us to simulate the approximate time for American reinforcements on this battlefield, let *C(t)* represent the following reinforcement schedule below as Engel uses for his calculations:

Table 2: The Battle of Iwo Jima (Engel) [E]

Day	**Troops Landed**
D+0	54,000
D+1	0
D+2	6,000
D+3	0
D+4	0
D+5	13,000

Engel's total number of troops landed at Iwo Jima: *73,000*

By using the tabulated data is input for the *C(t)* functions, Engel was able to derive, by a series of computations, the following values for *A* and *B* (See [E] for deeper mathematical evidence).

Engel's values	*A: 0.0544*	*B: 0.0106*

However, Engel's method of determining reinforcements has been debated by some scholars, including Colonel Robert W. Samz from the United States Military Academy, who challenged Engel's data on two major points: firstly, the reinforcement schedule, of which Samz's research shows the below result in contrast to Engel's table, and secondly, because of the different data on the reinforcement schedule, Samz found different values for his *A* and *B*.

Table 3: The Battle of Iwo Jima (Samz) [SR]

Day	Troops Landed
D+0	30,000
D+1	1,200
D+2	6,735
D+3	3,626
D+4	5,158
D+5	13,227
D+6	3,054
D+7	3,359
D+8	3,180
D+9	1,454
D+10	252

Samz's total number of troops landed at Iwo Jima: *71,245*

Examining Tables 2 and 3, we notice three important numerical differences:

1. The *total* number of troops landed has a difference of 1,755 troops, a 2.4% error on account of Engel's data.
2. The distribution of landing schedules each day maintains that D-day, D+2 day, and D+5 day are all peaks of the operation's troop landings, but according to Samz, only a little more than half of Engel's quoted initial D-day assault group landed on D-day.

   Samz insists that the entire landing operation for the United States Navy and Marine Corps combatants required an additional nine days after D-day, as opposed to Engel's four days.

Nevertheless, in keeping with Engel's notation, Colonel Samz reports that instead of taking the limit of the Lanchester differential equations, he uses a modified form of Theil's Inequality Coefficient. By some computations, Samz reaches this conclusion for values of *A* and *B*.

Samz's values	*A: 0.053996*	*B: 0.0119888*

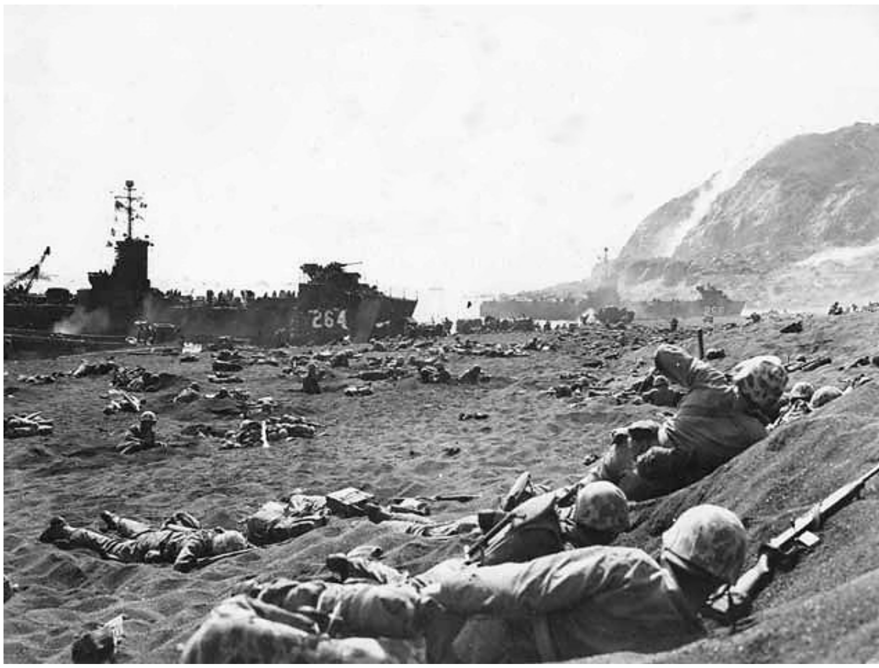


Fig. 6, Left: U.S. Marines take cover in the volcanic sand dunes on Iwo Jima as artillery hampers the beach assault [NPS]

Fig. 7, Right: The U.S. Flag flies over Mt. Suribachi, the volcanic peak of Iwo Jima. [NA]

Comparing Engel's and Samz's values of *A* and *B*, we see some negligible differences, and thus, this qualifies McKay's claim that Iwo Jima is one of the many battles which confirms that Lanchester's differential equations held true in the Second World War [MN 3].

How Convergences and Divergences Work on the Battlefield

"Forces operating on converging lines direct their action towards a common point; those operating on diverging lines do not. Now what are the effects of the action in the two cases? Here, we must seaparate tactics from strategy"

General Carl von Clausewitz, *Vom Krieg (On War)*, early 19th century [CC 3]

In the study of calculus, we observe that functions and sequences are either convergent, or divergent. Suppose we examine this from a sequential perspective because the applicable situation on the battlefield, as observed above, is measured in the number of battles, n. Then for a sequence x_n, it is said to be convergent if it approaches some limit [CN 1]. If our sequence x_n, is a battlefield maneuver, then the limit that x_n, approaches can represent the rally point of the forces that operate according to this sequence.

In land warfare, rally points are a physical locations where forces regroup to complete an objective. Flags, pennants, and guidons would be used in pre-20th century warfare, such that troops would move towards where the standards were located. We will further look into the modern definition of rally points, as a comparison to convergence limits in mathematics.

General Carl von Clausewitz mentions above that attack and defensive maneuvers can be thought of as movements toward or away from a point. Clausewitz devotes an entire chapter to exploring the idea of *"Convergence of attack and divergence of defence"* while never investigating the mathematical observations. However, Clausewitz recognizes the asymmetric relationship between attack and defense roles, and consequently, writes the following to define *defence:*

"What is the object of defence? To preserve. To preserve is easier than to acquire; from which follows at once that the means on both sides being supposed equal, the defensive is easier than the offensive...Every suspension of offensive action....is in favour of the side acting defensively." [CC 2]

Let us decompose the above information from Clausewitz such that the sequence x_n, is the core subject. That is, we are ignoring force structure, size, and effectiveness as well other circumstances, simply treating this sequence abstractly.

Suppose x_n, is a *defensive* movement, such that it fits Clausewitz's definition: *"In tactics every combat, great or small, is defensive if we leave the initiative to the enemy, and wait for his appearance in our front"* [CC 2]. Thus, since defense requires an objective that depends on the offense's movement, the defending forces are either moving very little, or not moving at all. However, when the offensive forces attack them, the defenders have the potential to be attacked on many sides. In fact, analyzing a single unprotected point on the terrain, there is 360° angle that this target is open to. That is, from any point on the ground close to the target, the attacker can thrust an offensive maneuver towards the target in any direction.

Therefore, we treat defensive maneuvers as divergent, because they have no clear direction or approach to fight in. While defenders may close the possible passages for an incoming offensive move, they are still uncertain about which avenue they should employ the most countermeasures in. Furthermore, sometimes, we cannot treat this in only a two-dimensional world, because in the cases of buildings, mountainous terrain, and other environments where height and depth can add more fighting positions, the enemy has even more choices to attack a defensive position. For this reason, air support gives its friendly forces on the ground an added advantage since it can seek, detect, and fight the enemy from the sky. We shall discuss air support more later, as it adds an advantage to ground forces whether or not they are attacking or defending [L 3].

Fig. 8, Above: Close Air Support (CAS) from an AH-64 Apache helicopter provides an overwatch for 3rd Infantry Division soldiers as they cross the Tigris River on a patrol. Such support systems from the air provide another dimension for defensive and counter-offensive measures [AG-USA]

Having shown that defenders can be thought of as performing a range of divergent sequences as maneuvers, we now move to show that attackers are performing convergent sequence movements. Suppose we are given an attacker's sequence of movements, y_n, $n \in \mathbb{N}$ such that $y_1, y_2, \ldots y_k$, are each bringing the attacker closer to the defending position or moving around it. Thus these moves are not displacing the attacker further away from the objective.

So let us call the objective O and basing concentric circles with centers at O then let $d(O, y_n)$ be the distance function between each sequence's final position and the objective O. Should we wish to represent this coordinate on the Cartesian plane, we will treat it as the origin, (0,0).

If, for each move $y_n \geq y_{n+1}$, then the function d represents the Euclidean distance between the attack group and the objective since $d(O,y_n)$ is a monotonic decreasing function, and it is bounded with the y_1 as an upper bound [K]. From the diagram below, we are working with concentric circles too, and thus, no negative scale exists, for once the attack sequence moves past the objective, it gain more distance from it, and from $y_n \geq y_{n+1}$, that cannot be possible since every point is closer to the point

O than the previous point in the sequence. Therefore, instead of remaining within the Cartesian system of positions (x,y), let us proceed to use polar coordinates, where for a position, (r, θ):

$$r = d(O, y_n) \text{ or } \quad r=\sqrt{(x^2+y^2)} \quad \text{and} \quad \theta=\tan^{-1}(\frac{y}{x})$$

With polar coordinates, we are now primarily concerned with the distance, expressed as the radius from a point to O. Perhaps the best method of visualizing this is to use the idea that we have a radar emplacement at O, and the radar at this objective scans every sector of the circle f times per minute, where f represents the frequency of a signal being sent out from the radar.

Therefore, the distance, or r, is measured by the ranging characteristics of the pulse sent out by the radar. By the sequential pulses, $r_1 \leq r_2 \leq \ldots \leq r_k = 0$ where the k^{th} range indicates the last pulse received by the radar—essentially when the attacking party is upon it. Thus, this shows that our y_k from the previous page is the final step in the sequence that shows that the sequence has converged upon O.

In the below diagram, Figure 9 illustrates a possible scenario for two attacking elements, *A1*, and *A2*, converging on *O.* Since by the above definition of "attacking" shows that both *A1* and *A2* need to not retrace their sequence to a larger circle, they must either continually move in to another concentric circle (which *A1* does every time) or they can remain at the same distance from *O*. The attack maneuver *A2* demonstrates this, as its third last leg does not change the distance from the origin, but you may notice that at some point between the beginning and end of this third leg, the distance function decreases, then increases again.

We can neglect instances such as this because each leg need not necessarily be a straight line from one concentric circle to the next inside it—these are simply directional bearings giving the overall displacement vector. For all we are concerned about, we simply know that from the i^{th} pulse to the $(i+1)^{th}$ pulse, the r value remains unchanged, but θ can change—remember that it need not, in which case, the maneuvere temporarily holds at its position for more than one pulse.

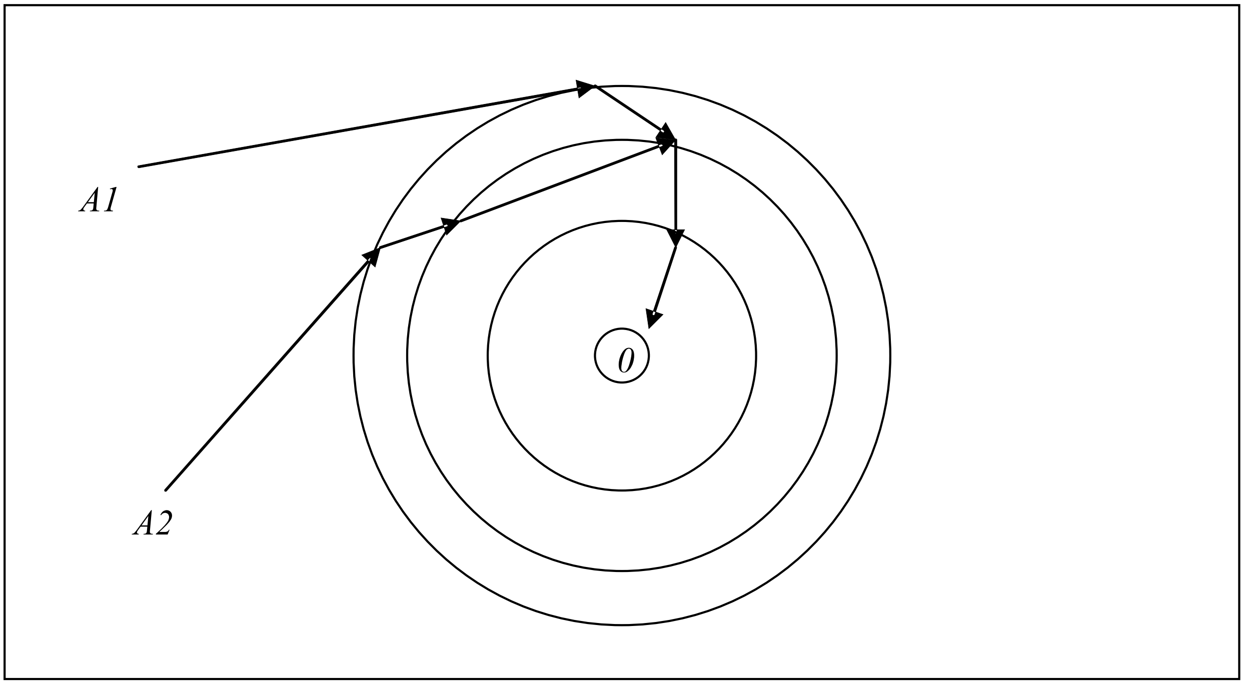

Fig. 9, Above: Two attacking maneuvers, A1 and A2, home in on the objective O. That is, their convergent attitude means that for every sequential move, A1 and A2 do not move further away from the target. In this way, their movement is always directed towards *O*.

Figure 9 above also allows for a special case of attacking maneuvers—the two elements of *A* meet at some point on the second largest concentric circle before completing the final two legs together. This specialty is observed on today's battlefield where, for instance, a laser-guided missile must follow a directed track to approach its target. In doing so, a spotter must use a laser designator, shown in Figure 5, to “paint” the target. Therefore, we can deduce from Figure 9 that if a laser designator is used to guide in two missiles, *A1* and *A2* then the designator is at least on or outside the second largest concentric circle, since that is the range *r* where both *A1* and *A2* meet their final approaches. For the spotter, this range, *r* should be greater than the minimum safe distance that friendly forces can be to the target *O*, since we wish not for friendly forces to be harmed by the collateral damage caused upon the implementation of *A1* and *A2.* Therefore, Clausewitz's idea of convergent attacks and divergent defences, can be shown to be complementary with the 21st century battlefield when we tinker with some polar geometry and the convergence of sequences.

The Fourth Arm: Lanchester's Insight to Aerial Warfare

Previously, it was mentioned that Close Air Support is beneficial to ground forces, and that the troops on the ground who have friends in the sky will be the ones that have the most hope to be victors. Lanchester's investigation into the age of aircraft entering warfare came at the early stages of propeller driven warplanes, and having had experience engineering automobiles, Lanchester took the same approach to analyzing aerial battles.

Lanchester immediately moved towards aviation warfare, with very little attention paid to the civil aviation side, especially because at the close of the 19th century, the modern machine gun had just been invented, and with the advent of the airplane, man bred both the machine gun and airplane together. Indeed, this would be one of Lanchester's greatest arguments into military science.

Even today, we speak of firepower in a number of ways, mainly caliber (size of round) and rate of fire (frequency of rounds fired). However, neither of these by themselves can give much indication as to how lethal a firearm is. Lanchester, having already worked with differential equations, used a new measure, simply titled, <u>Rapidity of Fire:</u>

Weight thrown per minute = nW_1 where W_1 is the mass of each round and

n is the number of rounds fired per minute.

The Rapidity of Fire is therefore, a combination of the rate of fire and caliber, such that we can gain a better perspective of how harmful a weapon can be. Lanchester's theory in aerial combat was borne upon the challenges already faced upon the battlefield, but Lanchester was more interested in the potential that fighter planes would have to engage with other fighter planes. Thus, most of his writing in *Aircraft in Warfare* concerns the fundamentals of air battles. Beginning with an analysis of the physics of the machine gun, Lanchester composes kinetic energy efficiency formulations and compares the different arms in the British arsenal to investigate into the best possible main weapon for British fighters [L 1]. Ultimately, the result that Lanchester reaches is this golden rule for armed combatants:

To be a successful combatant, the rate of fire must be one of the last things to compromise.

In addition to the offensive capability of an aircraft, Lanchester also observed that the same principles that apply to ground vehicles can be applied to aircraft too. For instance, it would be prudent to add more armor to a warplane since it will be vulnerable to gunfire. However, we want to maintain the same amount of maneuverability for the warplane such that it can match the same speed and turning radius as enemy planes. Since this agility is very important to retain, we must aim to balance the aircraft's mass accordingly, so in adding armor, some mass must be removed from the aircraft.

As a side note, a difference in aerodynamics *does* factor in to how maneuverable an aircraft is, as Lanchester studied, but according to the definition of Lanchester's laws, we are assuming similar forces—that is, the same technological capabilities are available to both sides, but that does not mean that they must employ such capabilities.

Therefore,if one is not to break Lanchester's rule for sacrificing the rate of fire, yet still desiring to decrease the weight of an aircraft to compensate for the additional kit of armor, the following choices remain:

- Decrease the warload of the aircraft (ammunition, ordnance, explosive compounds)
- Downsize the number of personnel onboard
- Decrease the powerplant's output

We can use a process of elimination here in finding that if one chooses to decrease the engine's power, that further slows down the aircraft and reduces the high-speed agility it needs to escape pursuit from the enemy aircraft. The only other two choices are to downsize the number of aircrew, in which case the workload on a bomber or fighter is reassigned to the remaining aircrew, but we immediately realize that this is not only impractical, it also only reduces a comparably small amount of weight compared to the armor that must surround the plane.

One may also argue that in today's battlespace, UAVs (Unmanned Aerial Vehicles) are an option, but in the event that we treat one of these aircraft to be a UAV, then little or no armor is necessary. Furthermore, UAVs are not used to dogfight, and are primarily used for reconnaissance and surveillance purposes.

This leaves only one option—decreasing the warload [L 2]. Lanchester concludes that this should be the only option that might be worth considering, while still giving the armored-up aircraft the advantage. His example is simple: Consider that a certain type of armor on the blue plane allows it to absorb 1200 rounds before going out of action, while the unarmored red plane absorbs 600 rounds before going out of action. Upon engaging with each other, the blue plane has the advantage of *survivability*. However, in order to gain, say, 30 pounds of armor, the blue plane must take 30 pounds less of ammunition of its original 60 pound payload, such that it doubles its protection, yet halves its rate of fire to conserve ammunition in a long dogfight.

Then this scenario is justified since for every two rounds that the blue plane receives, it discharges one round. On the flipside, for every round that the red plane receives, it discharges two rounds. Thus, a breakeven point occurs since the blue plane will eventually receive all 1200 hits from the red plane, and similarly, the red plane will receive all 600 hits from the blue plane, so both aircraft go out of action. This is the same scenario as when both aircraft were unarmored and carrying the same warload [L 2].

Thus, Lanchester's investigation found that this argument for armor can be quantitatively justified, and there even exists a category of armor that is worse than useless, where the armor is less than sufficient to protect the aircraft. That is, even with added armor, the rounds still penetrate the plane and aircraft may be better off without the added armor, because its survivability is enhanced with more agility.

Should one instead wish to investigate into the possibilities of the air combat theater and how each individual aircraft feeds into a success determined by a game theory approach, Berkovitz and Dreshner's research shows how an air war can be modeled. Although their work was completed in the 1960's, it is still prevalent today because the air combat theater has preserved two key components central to Berkovitz and Dreshner's work:

1. Two types of combat aircraft are available in a tactical air war—bombers and fighters
2. Both can be assigned to three different tasks:
    a) Counter air operations
    b) Air defense operations, and
    c) Ground support roles.

While Berkovitz and Dreshner appear to be less technical than Lanchester, one should bear in mind that they are not interested in how an individual warplane functions, but rather, how many warplanes function from two air forces to determine a superior airspace control [BD]. In turn, this means that the force that controls the skies can also provide ground support to its troops and thus, winning the air war provides an advantage for winning the ground war.

Conclusion

Since Lanchester's Law has been made known, it has been questioned many times upon the dawn of new technologies. Lanchester's work was primarily involved with armored cars and aircraft, yet both types of vehicles have evolved since—armored cars are not strong enough to survive IED blasts, and the warplanes that dominate today's skies pack a heavier payload, fly further, and rely on the support of other airplanes to support them.

However, the principles behind the 21st century weapons arsenal and defense systems remains unchanged. The overall objective and rationale in war has not transformed much, and the basic elements of an army, as mentioned in the introduction from Sun Tzu's work, are still present. We cannot do without the infantry or armored units in a war, as previous conflicts have shown. Air power cannot be enough to win a war, though as Lanchester predicted, they have changed the face of it. However, just as an attack helicopter cannot guard a cross roads, while a tank can, so too the new generations of robots and machines—they cannot guarantee a victory alone. Instead, we use what Lanchester gave us—combat models—to do exactly what they are meant for: modeling combat scenarios.

Indeed, the 21st century battlefield calls for perhaps the toughest situations yet, but we can be certain that Lanchester's combat models still provide a reliable guide to facing the uncertainties of warfare. Therefore, the 3:1 ratio of ground warfare is still preserved today as a general rule of thumb to predict whether an attacking force can overwhelm an enemy unit. We still see applications of Lanchester's laws in operations research, and since the publishing of *Aircraft in Warfare*, numerous organizations have flourished with the common goal of championing operations research to minimize casualties in wartime and peacetime.

At the time of this writing, military forces worldwide face the challenges of asymmetric warfare, but even so, Lanchester's guerrilla models have been able to account for such types of unconventional warfare. We also witness advances in the unmanned technologies, with robots replacing humans in many roles ranging from surveillance to bomb-inspection.

Furthermore, one can be convinced of Lanchester's ordinary differential equations still being powerful today because of the nature of attrition—it is everywhere! Moreover, the legions depend on a finite amount of resources and the will of fighting men, both of which will wane over time. Simply put, Lanchester's Laws are still relevant today because the process of decay excludes none who live.

Therefore, the last century and its worth of research in military sciences has perhaps been no greater than the ages beforehand in the days of Machiavelli and Clausewitz, and in every generation, it is hoped that they learn from mistakes of the past. In closing, we consider perhaps the strongest argument to why Lanchester's view of ground forces and their support will still prevail into this century and beyond when war is the last alternative in conflicts:

"You may fly over a land forever; you may bomb it, atomize it, pulverize it and wipe it clean of life—but if you desire to defend it, protect it, and keep it for civilization, you must do this on the ground, the way the Roman legions did, by putting your young men into the mud."

T. R Fehrenbach, *This Kind of War,* 1963 [F]

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